# Grade 4 Accelerated B.E.S.T. Instructional Guide for Mathematics

The B.E.S.T. Instructional Guide for Mathematics (B1G-M) is intended to assist educators with planning for student learning and instruction aligned to Florida’s Benchmarks for Excellent Student Thinking (B.E.S.T.) Standards. This guide is designed to aid high-quality instruction through the identification of components that support the learning and teaching of the B.E.S.T. Mathematics Standards and Benchmarks. The B1G-M includes an analysis of information related to the B.E.S.T. Standards for Mathematics within this specific mathematics course, the instructional emphasis and aligned resources. This document is posted on the [B.E.S.T. Standards for Mathematics webpage](https://www.fldoe.org/academics/standards/subject-areas/math-science/mathematics/bestmath.stml) of the Florida Department of Education’s website and will continue to undergo edits as needed.

## Structural Framework and Intentional Design of the B.E.S.T. Standards for Mathematics

Florida’s B.E.S.T. Standards for Mathematics were built on the following.

* The coding scheme for the standards and benchmarks was changed to be consistent with other content areas. The new coding scheme is structured as follows:

Content.GradeLevel.Strand.Standard.Benchmark.

* Strands were streamlined to be more consistent throughout.
* The standards and benchmarks were written to be clear and concise to ensure that they are easily understood by all stakeholders.
* The benchmarks were written to allow teachers to meet students’ individual skills, knowledge and ability.
* The benchmarks were written to allow students the flexibility to solve problems using a method or strategy that is accurate, generalizable and efficient depending on the content (i.e., the numbers, expressions or equations).
* The benchmarks were written to allow for student discovery (i.e., exploring) of strategies rather than the teaching, naming and assessing of each strategy individually.
* The benchmarks were written to support multiple pathways for success in career and college for students.
* The benchmarks should not be taught in isolation but should be combined purposefully.
* The benchmarks may be addressed at multiple points throughout the year, with the intention of gaining mastery by the end of the year.
* Appropriate progression of content within and across strands was developed for each grade level and across grade levels.
* There is an intentional balance of conceptual understanding and procedural fluency with the application of accurate real-world context intertwined within mathematical concepts for relevance.
* The use of other content areas, like science and the arts, within real-world problems should be accurate, relevant, authentic and reflect grade-level appropriateness.

## Components of the B.E.S.T. Instructional Guide for Mathematics

The following table is an example of the layout for each benchmark and includes the defining attributes for each component. It is important to note that instruction should not be limited to the possible connecting benchmarks, related terms, strategies or examples provided. To do so would strip the intention of an educator meeting students’ individual skills, knowledge and abilities.

Benchmark

focal point for instruction within lesson or task

This section includes the benchmark as identified in the [B.E.S.T. Standards for Mathematics](https://www.cpalms.org/uploads/docs/standards/BEST/MA/MathBESTStandardsFinal.pdf). The benchmark, also referred to as the Benchmark of Focus, is the focal point for student learning and instruction. The benchmark, and its related example(s) and clarification(s), can also be found in the course description. The 9-12 benchmarks may be included in multiple courses; select the example(s) or clarification(s) as appropriate for the identified course.

Connecting Benchmarks/Horizontal Alignment

in other standards within the grade level or course

This section includes a list of connecting benchmarks that relate horizontally to the Benchmark of Focus. Horizontal alignment is the intentional progression of content within a grade level or course linking skills within and across strands. Connecting benchmarks are benchmarks that either make a mathematical connection or include prerequisite skills. The information included in this section is not a comprehensive list, and educators are encouraged to find other connecting benchmarks. Additionally, this list will not include benchmarks from the same standard since benchmarks within the same standard already have an inherent connection.

Terms from the K-12 Glossary

This section includes terms from Appendix C: K-12 Glossary, found within the B.E.S.T. Standards for Mathematics document, which are relevant to the identified Benchmark of Focus. The terms included in this section should not be viewed as a comprehensive vocabulary list, but instead should be considered during instruction or act as a reference for educators.

Vertical Alignment

across grade levels or courses

This section includes a list of related benchmarks that connect vertically to the Benchmark of Focus. Vertical alignment is the intentional progression of content from one year to the next, spanning across multiple grade levels. Benchmarks listed in this section make mathematical connections from prior grade levels or courses in future grade levels or courses within and across strands. If the Benchmark of Focus is a new concept or skill, it may not have any previous benchmarks listed. Likewise, if the Benchmark of Focus is a mathematical skill or concept that is finalized in learning and does not have any direct connection to future grade levels or courses, it may not have any future benchmarks listed. The information included in this section is not a comprehensive list, and educators are encouraged to find other benchmarks within a vertical progression.

Purpose and Instructional Strategies

This section includes further narrative for instruction of the benchmark and vertical alignment. Additionally, this section may also include the following:

* explanations and details for the benchmark;
* vocabulary not provided within Appendix C;
* possible instructional strategies and teaching methods; and
* strategies to embed potentially related Mathematical Thinking and Reasoning Standards (MTRs).

Common Misconceptions or Errors

This section will include common student misconceptions or errors and may include strategies to address the identified misconception or error. Recognition of these misconceptions and errors enables educators to identify them in the classroom and make efforts to correct the misconception or error. This corrective effort in the classroom can also be a form of formative assessment within instruction.

**Strategies** to Support Tiered Instruction

The instructional strategies in this section address the common misconceptions and errors listed within the above section that can be a barrier to successfully learning the benchmark. All instruction and intervention at Tiers 2 and 3 are intended to support students to be successful with Tier 1 instruction. Strategies that support tiered instruction are intended to assist teachers in planning across any tier of support and should not be considered exclusive or inclusive of other instructional strategies that may support student learning with the B.E.S.T. Mathematics Standards. For more information about tiered instruction, please see the Effective Tiered Instruction for Mathematics: ALL Means ALL document.

Instructional Tasks

demonstrate the depth of the benchmark and the connection to the related benchmarks

This section will include example instructional tasks, which may be open-ended and are intended to demonstrate the depth of the benchmark. Some instructional tasks include integration of the Mathematical Thinking and Reasoning Standards (MTRs) and related benchmark(s). Enrichment tasks may be included to make connections to benchmarks in later grade levels or courses. Tasks may require extended time, additional materials and collaboration.

Instructional Items

demonstrate the focus of the benchmark

This section will include example instructional items which may be used as evidence to demonstrate the students’ understanding of the benchmark. Items may highlight one or more parts of the benchmark.

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

# Mathematical Thinking and Reasoning Standards

MTRs: Because Math Matters

Florida students are expected to engage with mathematics through the Mathematical Thinking and Reasoning Standards (MTRs) by utilizing their language as a self-monitoring tool in the classroom, promoting deeper learning and understanding of mathematics. The MTRs are standards which should be used as a lens when planning for student learning and instruction of the B.E.S.T. Standards for Mathematics.

## Structural Framework and Intentional Design of the Mathematical Thinking and Reasoning Standards

The Mathematical Thinking and Reasoning Standards (MTRs) are built on the following.

* The MTRs have the same coding scheme as the standards and benchmarks; however, they are written at the standard level because there are no benchmarks.
* In order to fulfill Florida’s unique coding scheme, the 5th place (benchmark) will always be a “1” for the MTRs.
* The B.E.S.T. Standards for Mathematics should be taught through the lens of the MTRs.
* At least one of the MTRs should be authentically and appropriately embedded throughout every lesson based on the expectation of the benchmark(s).
* The bulleted language of the MTRs were written for students to use as self-monitoring tools during daily instruction.
* The clarifications of the MTRs were written for teachers to use as a guide to inform their instructional practices.
* The MTRs ensure that students stay engaged, persevere in tasks, share their thinking, balance conceptual understanding and procedures, assess their solutions, make connections to previous learning and extended knowledge, and apply mathematical concepts to real-world applications.
* The MTRs should not stand alone as a separate focus for instruction but should be combined purposefully.
* The MTRs will be addressed at multiple points throughout the year, with the intention of gaining mastery of mathematical skills by the end of the year and building upon these skills as they continue in their K-12 education.

MA.K12.MTR.1.1 Actively participate in effortful learning both individually and collectively.

Mathematicians who participate in effortful learning both individually and with others:

* Analyze the problem in a way that makes sense given the task.
* Ask questions that will help with solving the task.
* Build perseverance by modifying methods as needed while solving a challenging task.
* Stay engaged and maintain a positive mindset when working to solve tasks.
* Help and support each other when attempting a new method or approach.

Clarifications:

Teachers who encourage students to participate actively in effortful learning both individually and with others:

* Cultivate a community of growth mindset learners.
* Foster perseverance in students by choosing tasks that are challenging.
* Develop students’ ability to analyze and problem solve.
* Recognize students’ effort when solving challenging problems.

MA.K12.MTR.2.1 Demonstrate understanding by representing problems in multiple ways.

Mathematicians who demonstrate understanding by representing problems in multiple ways:

* Build understanding through modeling and using manipulatives.
* Represent solutions to problems in multiple ways using objects, drawings, tables, graphs and equations.
* Progress from modeling problems with objects and drawings to using algorithms and equations.
* Express connections between concepts and representations.
* Choose a representation based on the given context or purpose.

Clarifications:

Teachers who encourage students to demonstrate understanding by representing problems in multiple ways:

* Help students make connections between concepts and representations.
* Provide opportunities for students to use manipulatives when investigating concepts.
* Guide students from concrete to pictorial to abstract representations as understanding progresses.
* Show students that various representations can have different purposes and can be useful in different situations.

MA.K12.MTR. 3.1 Complete tasks with mathematical fluency.

Mathematicians who complete tasks with mathematical fluency:

* Select efficient and appropriate methods for solving problems within the given context.
* Maintain flexibility and accuracy while performing procedures and mental calculations.
* Complete tasks accurately and with confidence.
* Adapt procedures to apply them to a new context.
* Use feedback to improve efficiency when performing calculations.

Clarifications:

Teachers who encourage students to complete tasks with mathematical fluency:

* Provide students with the flexibility to solve problems by selecting a procedure that allows them to solve efficiently and accurately.
* Offer multiple opportunities for students to practice efficient and generalizable methods.
* Provide opportunities for students to reflect on the method they used and determine if a more efficient method could have been used.

MA.K12.MTR.4.1 Engage in discussions that reflect on the mathematical thinking of self and others.

Mathematicians who engage in discussions that reflect on the mathematical thinking of self and others:

* Communicate mathematical ideas, vocabulary and methods effectively.
* Analyze the mathematical thinking of others.
* Compare the efficiency of a method to those expressed by others.
* Recognize errors and suggest how to correctly solve the task.
* Justify results by explaining methods and processes.
* Construct possible arguments based on evidence.

Clarifications:

Teachers who encourage students to engage in discussions that reflect on the mathematical thinking of self and others:

* Establish a culture in which students ask questions of the teacher and their peers, and error is an opportunity for learning.
* Create opportunities for students to discuss their thinking with peers.
* Select, sequence and present student work to advance and deepen understanding of correct and increasingly efficient methods.
* Develop students’ ability to justify methods and compare their responses to the responses of their peers.

MA.K12.MTR.5.1 Use patterns and structure to help understand and connect mathematical concepts.

Mathematicians who use patterns and structure to help understand and connect mathematical concepts:

* Focus on relevant details within a problem.
* Create plans and procedures to logically order events, steps or ideas to solve problems.
* Decompose a complex problem into manageable parts.
* Relate previously learned concepts to new concepts.
* Look for similarities among problems.
* Connect solutions of problems to more complicated large-scale situations.

Clarifications:

Teachers who encourage students to use patterns and structure to help understand and connect mathematical concepts:

* Help students recognize the patterns in the world around them and connect these patterns to mathematical concepts.
* Support students to develop generalizations based on the similarities found among problems.
* Provide opportunities for students to create plans and procedures to solve problems.
* Develop students’ ability to construct relationships between their current understanding and more sophisticated ways of thinking.

MA.K12.MTR.6.1 Assess the reasonableness of solutions.

Mathematicians who assess the reasonableness of solutions:

* Estimate to discover possible solutions.
* Use benchmark quantities to determine if a solution makes sense.
* Check calculations when solving problems.
* Verify possible solutions by explaining the methods used.
* Evaluate results based on the given context.

Clarifications:

Teachers who encourage students to assess the reasonableness of solutions:

* Have students estimate or predict solutions prior to solving.
* Prompt students to continually ask, “Does this solution make sense? How do you know?”
* Reinforce that students check their work as they progress within and after a task.
* Strengthen students’ ability to verify solutions through justifications.

MA.K12.MTR.7.1 Apply mathematics to real-world contexts.

Mathematicians who apply mathematics to real-world contexts:

* Connect mathematical concepts to everyday experiences.
* Use models and methods to understand, represent and solve problems.
* Perform investigations to gather data or determine if a method is appropriate.
* Redesign models and methods to improve accuracy or efficiency.

Clarifications:

Teachers who encourage students to apply mathematics to real-world contexts:

* Provide opportunities for students to create models, both concrete and abstract, and perform investigations.
* Challenge students to question the accuracy of their models and methods.
* Support students as they validate conclusions by comparing them to the given situation.
* Indicate how various concepts can be applied to other disciplines.

## Examples of Teacher and Student Moves for the MTRs

Below are examples that demonstrate the embedding of the MTRs within the mathematics classroom. The provided teacher and student moves are examples of how some MTRs could be incorporated into student learning and instruction. The information included in this table is not a comprehensive list, and educators are encouraged to incorporate other teacher and student moves that support the MTRs.

| **MTR** | **Student Moves** | **Teacher Moves** |
| --- | --- | --- |
| MA.K12.MTR.1.1  *Actively participate in effortful learning both individually and collectively.* | * Students engage in the task through individual analysis, student-to-teacher interaction and student-to-student interaction. * Students ask task-appropriate questions to self, the teacher and to other students. *(MTR.4.1)* * Students have a positive productive struggle exhibiting growth mindset, even when making a mistake. * Students stay engaged in the task to a purposeful conclusion while modifying methods, when necessary, in solving a problem through self-analysis and perseverance. | * Teacher provides flexible options (i.e., differentiated, challenging tasks that allow students to actively pursue a solution both individually and in groups) so that all students have the opportunity to access and engage with instruction, as well as demonstrate their learning. * Teacher creates a physical environment that supports a growth mindset and will ensure positive student engagement and collaboration. * Teacher provides constructive, encouraging feedback to students that recognizes their efforts and the value of analysis and revision. * Teacher provides appropriate time for student processing, productive struggle and reflection. * Teacher uses data and questions to focus students on their thinking; help students determine their sources of struggle and to build understanding. * Teacher encourages students to ask appropriate questions of other students and of the teacher including questions that examine accuracy. *(MTR.4.1)* |
| MA.K12.MTR.2.1  *Demonstrate understanding by representing problems in multiple ways.* | * Students represent problems concretely using objects, models and manipulatives. * Students represent problems pictorially using drawings, models, tables and graphs. * Students represent problems abstractly using numerical or algebraic expressions and equations. * Students make connections and select among different representations and methods for the same problem, as appropriate to different situations or context. *(MTR.3.1)* | * Teacher provides students with objects, models, manipulatives, appropriate technology and real-world situations. *(MTR.7.1)* * Teacher encourages students to use drawings, models, tables, expressions, equations and graphs to represent problems and solutions. * Teacher questions students about making connections between different representations and methods and challenges students to choose one that is most appropriate to the context. *(MTR.3.1)* * Teacher encourages students to explain their different representations and methods to each other. *(MTR.4.1)* * Teacher provides opportunities for students to choose appropriate methods and to use mathematical technology. |
| MA.K12.MTR.3.1  *Complete tasks with mathematical fluency.* | * Students complete tasks with flexibility, efficiency and accuracy. * Students use feedback from peers and teachers to reflect on and revise methods used. * Students build confidence through practice in a variety of contexts and problems. *(MTR.1.1)* | * Teacher provides tasks and opportunities to explore and share different methods to solve problems. *(MTR.1.1)* * Teacher provides opportunities for students to choose methods and reflect (i.e., through error analysis, revision, summarizing methods or writing) on the efficiency and accuracy of the method(s) chosen. * Teacher asks questions and gives feedback to focus student thinking to build efficiency of accurate methods. * Teacher offers multiple opportunities to practice generalizable methods. |
| MA.K12.MTR.4.1  *Engage in discussions that reflect on the mathematical thinking of self and others.* | * Students use content specific language to communicate and justify mathematical ideas and chosen methods. * Students use discussions and reflections to recognize errors and revise their thinking. * Students use discussions to analyze the mathematical thinking of others. * Students identify errors within their own work and then determine possible reasons and potential corrections. * When working in small groups, students recognize errors of their peers and offer suggestions. | * Teacher provides students with opportunities (through open-ended tasks, questions and class structure) to make sense of their thinking. *(MTR.1.1)* * Teacher uses precise mathematical language, both written and abstract, and encourages students to revise their language through discussion. * Teacher creates opportunities for students to discuss and reflect on their choice of methods, their errors and revisions and their justifications. * Teachers select, sequence and present student work to elicit discussion about different methods and representations. *(MTR.2.1, MTR.3.1)* |
| MA.K12.MTR.5.1 *Use patterns and structure to help understand and connect mathematical concepts.* | * Students identify relevant details in a problem in order to create plans and decompose problems into manageable parts. * Students find similarities and common structures, or patterns, between problems in order to solve related and more complex problems using prior knowledge. | * Teacher asks questions to help students construct relationships between familiar and unfamiliar problems and to transfer this relationship to solve other problems. *(MTR.1.1)* * Teacher provides students opportunities to connect prior and current understanding to new concepts. * Teacher provides opportunities for students to discuss and develop generalizations about a mathematical concept. *(MTR.3.1, MTR.4.1)* * Teacher allows students to develop an appropriate sequence of steps in solving problems. * Teacher provides opportunities for students to reflect during problem solving to make connections to problems in other contexts, noticing structure and making improvements to their process. |
| MA.K12.MTR.6.1 *Assess the reasonableness of solutions.* | * Students estimate a solution, including using benchmark quantities in place of the original numbers in a problem. * Students monitor calculations, procedures and intermediate results during the process of solving problems. * Students verify and check if solutions are viable, or reasonable, within the context or situation. *(MTR.7.1)* * Students reflect on the accuracy of their estimations and their solutions. | * Teacher provides opportunities for students to estimate or predict solutions prior to solving. * Teacher encourages students to compare results to estimations and revise if necessary for future situations. *(MTR.5.1)* * Teacher prompts students to self-monitor by continually asking, “Does this solution or intermediate result make sense? How do you know?” * Teacher encourages students to provide explanations and justifications for results to self and others. *(MTR.4.1)* |
| MA.K12.MTR.7.1 *Apply mathematics to real-world contexts.* | * Student Students connect mathematical concepts to everyday experiences. * Students use mathematical models and methods to understand, represent and solve real-world problems. * Students investigate, research and gather data to determine if a mathematical model is appropriate for a given situation from the world around them. * Students re-design models and methods to improve accuracy or efficiency. | * Teacher provides real-world context to help students build understanding of abstract mathematical ideas. * Teacher encourages students to assess the validity and accuracy of mathematical models and situations in real-world context, and to revise those models if necessary. * Teacher provides opportunities for students to investigate, research and gather data to determine if a mathematical model is appropriate for a given situation from the world around them. * Teacher provides opportunities for students to apply concepts to other content areas. |

# Grade 4 Accelerated Areas of Emphasis

In Grade 4 Accelerated, instructional time will emphasize five areas:

(1) developing the relationship between fractions and decimals;

(2) multiplying and dividing multi-digit whole numbers, including using a standard algorithm;

(3) adding and subtracting fractions and decimals with procedural fluency, developing an understanding of multiplication and division of fractions and decimals;

(4) developing an understanding of the coordinate plane and plotting pairs of numbers in the first quadrant;

(5) extending geometric reasoning to include volume and

(6) developing an understanding for interpreting data to include mean, mode, median and range.

The purpose of the areas of emphasis is not to guide specific units of learning and instruction, but rather provide insight on major mathematical topics that will be covered within this mathematics course. In addition to its purpose, the areas of emphasis are built on the following.

* Supports the intentional horizontal progression within the strands and across the strands in this grade level or course.
* Student learning and instruction should not focus on the stated areas of emphasis as individual units.
* Areas of emphasis are addressed within standards and benchmarks throughout the course so that students are making connections throughout the school year.
* Some benchmarks can be organized within more than one area.
* Supports the communication of the major mathematical topics to all stakeholders.
* Benchmarks within the areas of emphasis should not be taught within the order in which they appear. To do so would strip the progression of mathematical ideas and miss the opportunity to enhance horizontal progressions within the grade level or course.

The table below shows how the benchmarks within this mathematics course are embedded within the areas of emphasis.

| **Number Sense and Operations** | Relationship between fractions and decimals | Multiplying and dividing multi-digit whole numbers | Adding and subtracting fractions and decimals | Understanding of the coordinate plane | Extending geometric reasoning to include volume | Developing an understanding for interpreting data |
| --- | --- | --- | --- | --- | --- | --- |
| [MA.4.NSO.1.1](#_MA.4.NSO.1.1_&_MA.5.NSO.1.1) |  | X |  |  |  |  |
| [MA.4.NSO.1.5](#_MA.4.NSO.1.5) | X |  |  |  |  |  |
| [MA.4.NSO.2.3](#_MA.4.NSO.2.3_&_MA.5.NSO.2.1) |  | X |  |  |  |  |
| [MA.4.NSO.2.4](#_MA.4.NSO.2.4_&_5.NSO.2.2) |  | X |  |  |  |  |
| [MA.4.NSO.2.6](#_MA.4.NSO.2.6) | X | X |  |  |  |  |
| [MA.4.NSO.2.7](#_MA.4.NSO.2.7_&_MA.5.NSO.2.3) | X |  |  |  |  |  |
| [MA.5.NSO.1.1](#_MA.6.NSO.1.1) |  |  | X |  |  |  |
| [MA.5.NSO.1.2](#_MA.5.NSO.1.2) |  |  | X |  |  |  |
| [MA.5.NSO.1.3](#_MA.5.NSO.1.3) |  |  | X |  |  |  |
| [MA.5.NSO.1.4](#_MA.5.NSO.1.4) |  |  | X |  |  |  |
| [MA.5.NOS.1.5](#_MA.5.NSO.1.5) |  |  | X |  |  |  |
| [MA.5.NSO.2.1](#_MA.4.NSO.2.3_&_MA.5.NSO.2.1_1) |  | X |  |  |  |  |
| [MA.5.NSO.2.2](#_MA.4.NSO.2.4_&_MA.5.NSO.2.2) |  | X |  |  |  |  |
| [MA.5.NSO.2.3](#_MA.5.NSO.2.3) |  |  | X |  |  |  |
| [MA.5.NSO.2.4](#_MA.5.NSO.2.4) |  |  | X |  |  |  |
| [MA.5.NSO.2.5](#_MA.5.NSO.2.5) |  |  | X |  |  |  |

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| --- | --- | --- | --- | --- | --- |
| **Fractions** | Relationship between fractions and decimals | Multiplying and dividing multi-digit whole numbers | Adding and subtracting fractions and decimals | Understanding of the coordinate plane | Extending geometric reasoning to include volume |
| [MA.4.FR.1.2](#_MA.4.FR.1.2) | X |  |  |  |  |
| [MA.4.FR.2.4](#_MA.4.FR.2.4_&_MA.5.FR.2.2) | X | X |  |  |  |
| [MA.5.FR.1.1](#_MA.5.FR.1.1) |  |  | X |  |  |
| [MA.5.FR.2.1](#_MA.5.FR.2.1) |  |  | X |  |  |
| [MA.5.FR.2.2](#_MA.4.FR.2.4_&_MA.5.FR.2.2) |  |  | X |  |  |
| [MA.5.FR.2.3](#_MA.5.FR.2.3) |  |  | X |  |  |
| [MA.5.FR.2.4](#_MA.5.FR.2.4) |  |  | X |  |  |

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| --- | --- | --- | --- | --- | --- |
| **Algebraic Reasoning** | Relationship between fractions and decimals | Multiplying and dividing multi-digit whole numbers | Adding and subtracting fractions and decimals | Understanding of the coordinate plane | Extending geometric reasoning to include volume |
| [MA.4.AR.1.1](#_MA.4.AR.1.1) |  | X |  |  |  |
| [MA.4.AR.1.3](#_MA.4.AR.1.3_&_MA.5.AR.1.2) | X | X | X |  |  |
| [MA.5.AR.1.1](#_MA.5.AR.1.1) |  | X |  |  |  |
| [MA.5.AR.1.2](#_MA.4.AR.1.2) |  |  |  |  |  |
| [MA.5.AR.1.3](#_MA.5.AR.1.3) |  |  |  |  |  |
| [MA.5.AR.2.1](#_MA.5.AR.2.1) |  | X | X |  |  |
| [MA.5.AR.2.2](#_MA.5.AR.2.2) |  | X | X |  |  |
| [MA.5.AR.2.3](#_MA.5.AR.2.3) |  | X |  |  |  |
| [MA.5.AR.2.4](#_MA.5.AR.2.4) |  | X |  |  |  |
| [MA.5.AR.3.1](#_MA.5.AR.3.1) |  | X |  |  |  |
| [MA.5.AR.3.2](#_MA.5.AR.3.2) |  | X |  |  |  |

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| --- | --- | --- | --- | --- | --- |
| **Measurement** | Relationship between fractions and decimals | Multiplying and dividing multi-digit whole numbers | Adding and subtracting fractions and decimals | Understanding of the coordinate plane | Extending geometric reasoning to include volume |
| [MA.4.M.1.1](#_MA.4.M.1.1) |  |  |  |  |  |
| [MA.4.M.1.2](#_MA.4.M.1.2) |  | X |  |  |  |
| [MA.4.M.2.1](#_MA.4.M.2.1) | X | X |  |  |  |
| [MA.4.M.2.2](#_MA.4.M.2.2) | X |  |  |  |  |
| [MA.5.M.1.1](#_MA.5.M.1.1) |  | X | X |  | X |
| [MA.5.M.2.1](#_MA.5.M.2.1) |  | X | X |  | X |

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| --- | --- | --- | --- | --- | --- |
| **Geometric Reasoning** | Relationship between fractions and decimals | Multiplying and dividing multi-digit whole numbers | Adding and subtracting fractions and decimals | Understanding of the coordinate plane | Extending geometric reasoning to include volume |
| [MA.5.GR.1.1](#_MA.5.GR.1.1) |  |  |  |  | X |
| [MA.5.GR.1.2](#_MA.5.GR.1.2) |  |  |  |  | X |
| [MA.5.GR.2.1](#_MA.5.GR.2.1) |  |  | X |  | X |
| [MA.5.GR.3.1](#_MA.5.GR.3.1) |  |  |  |  | X |
| [MA.5.GR.3.2](#_MA.5.GR.3.2) |  |  |  |  | X |
| [MA.5.GR.3.3](#_MA.5.GR.3.3) |  |  |  |  | X |
| [MA.5.GR.4.1](#_MA.5.GR.4.1) |  |  |  | X |  |
| [MA.5.GR.4.2](#_MA.5.GR.4.2) |  |  |  | X |  |

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| --- | --- | --- | --- | --- | --- |
| **Data Analysis & Probability** | Relationship between fractions and decimals | Multiplying and dividing multi-digit whole numbers | Adding and subtracting fractions and decimals | Understanding of the coordinate plane | Extending geometric reasoning to include volume |
| [MA.4.DP.1.1](#_MA.4.DP.1.1_&_MA.5.DP.) | X |  |  |  |  |
| [MA.4.DP.1.2](#_MA.4.DP.1.2_&_MA.5.DP.1.2) | X |  |  |  |  |
| [MA.4.DP.1.3](#_MA.4.DP.1.3) | X |  |  |  |  |
| [MA.5.DP.1.1](#_MA.4.DP.1.1) | X |  |  |  |  |
| [MA.5.DP.1.2](#_MA.4.DP.1.2) | X |  |  |  |  |

## Number Sense and Operations

**MA.4.NSO.1** *Understand the place value of multi-digit numbers with decimals to the thousandth place.*

### MA.4.NSO.1.1 & MA.5.NSO.1.1

Benchmark (Grade 4)

MA.4.NSO.1.1 Express how the value of a digit in a multi-digit whole number changes if the digit moves one place to the left or right

Benchmark (Grade 5)

MA.5.NSO.1.1 Express how the value of a digit in a multi-digit number with decimals to the thousandths changes if the digit moves one or more places to the left or right.

Connecting Benchmarks/Horizontal Alignment

* MA.4.NSO.2.5
* MA.5.NSO.2.4, MA.5.NSO.2.5
* MA.5.AR.2.1, MA.5.AR.2.2, MA.5.AR.2.3
* MA.5.M.1.1
* MA.5.M.2.1

Terms from the K-12 Glossary

* Whole Number

Vertical Alignment

|  |  |
| --- | --- |
| **Previous Benchmarks**   * MA.3.NSO.2.3 | **Next Benchmarks**   * MA.6.NSO.1.1 |

Purpose and Instructional Strategies Integers

The purpose of this benchmark is to extend students’ understanding of place value to build a foundation for multiplying and dividing by 10. Students should work with the idea that the tens place is ten times as much as the ones place, and the ones place is the size of the tens place. Students use these patterns as they generalize place value relationships with decimals *(MTR.5.1)*.

Throughout instruction, teachers should have students practice this concept using place value charts, base-ten blocks and/or digit cards to manipulate and investigate place value relationships.

Table, timeline


* In the Accelerated Grade 4 course, this benchmark is taught in conjunction with MA.4.NSO.1.1. In MA.5.NSO.1.1, the focus is on that a digit in one place represents 10 times what it represents in the place to its right and of what it represents in the place to its left. All of this work forms the foundation for arithmeticand algorithms with decimals which is completed in Grade 6 (MA.6.NSO.2.1).
* To help students understand the meaning of the and  *of* relationship, students can use base ten manipulatives or simply bundle classroom objects (e.g., paper clips, pretzel sticks). Students should name numbers and use verbal descriptions to explain the relationship between numbers (e.g., “6 is 10 times greater than 6 tenths, and 6 tenths is of 6”). In addition to physical manipulatives, place value charts help students understand the relationship between digits in different places *(MTR.2.1)*.
* Instruction of this benchmark should connect with student work with whole numbers. For example, students who understand can reason that because is of , therefore its product with 2 will be of *(MTR.5.1)*.
* Instruction builds on the patterns of multiplying by 10 and , and extends to multiplying by values such as 100 and that will cause the digits to shift more than one place to the left and right. (MTR.5.1)

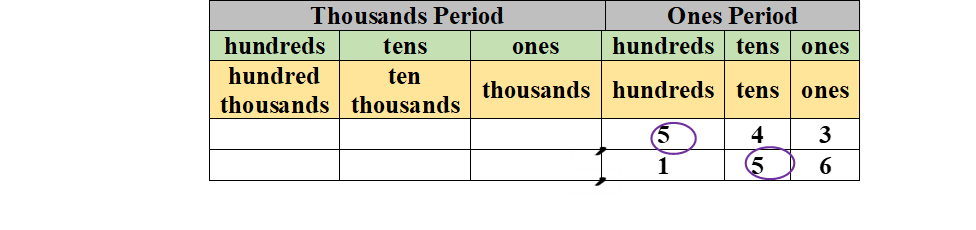
Common Misconceptions or Errors

* Students do not understand that when the digit moves to the left that it has increased a place value which is the same thing as multiplying by 10 and when the digit moves to the right that is has decreased a place value, which is the same thing as dividing by 10. It is important to have math discourse throughout instruction about why this is happening.
* Students do not understand that the value of a digit is 10 times the value of the digit to its right only if the digits are the same.
* Students can misunderstand what “ of” a number represents. Teachers can connect of to “ten times less” or “dividing by 10” to help students connect of a number to 10 times greater.
* Students who use either rule “move the decimal point” or “shift the digits” without understanding when multiplying by a power of ten can easily make errors. Students need to understand that from either point of view, the position of the decimal point marks the transition between the ones and the tenths place.

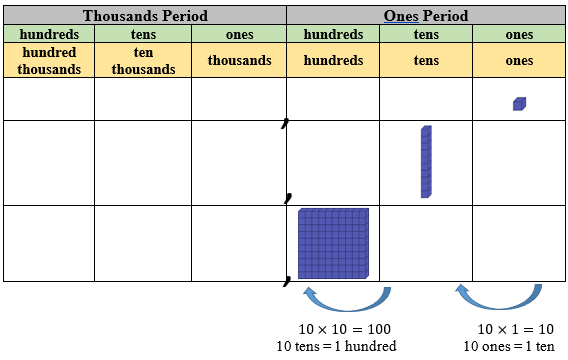
Strategies to Support Tiered Instruction

Instruction includes opportunities to use a place value chart and manipulatives such as base-ten blocks to demonstrate how the value of a digit changes if the digit moves one place to the left or right. Have math discourse throughout instruction about why this is happening.

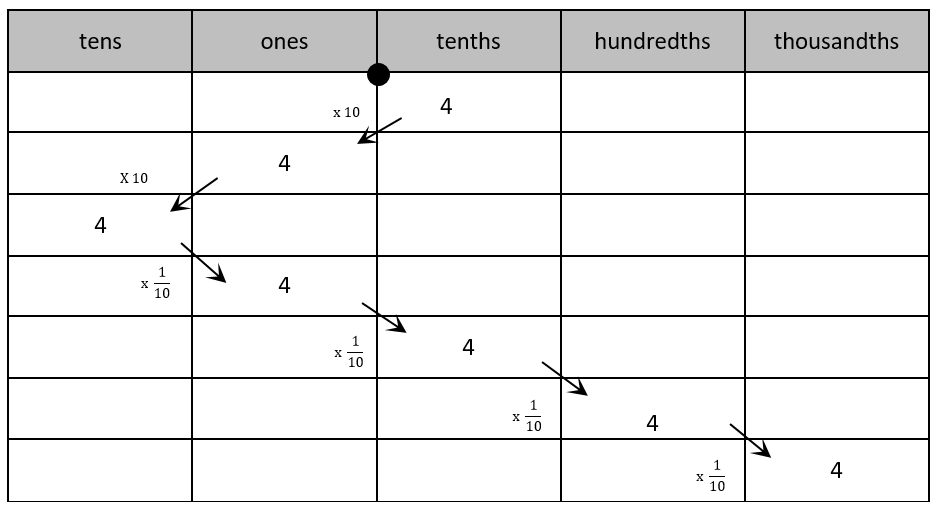
* For example, the 5 in 543 is 10 times greater than the 5 in 156. Students write 543 and 156 in a place value chart like the one shown below and compare the value of the 5’s (500 and 50) using the place value charts and equations. The teacher explains that the 5 in the hundreds place represents the value 500, which is 10 times greater than the value 50 represented by the 5 in the tens place. Use a place value chart to show this relationship while writing the equation to reinforce this relationship. The teacher explains that the 5 in the tens place represents the value 50, which is 10 times less than the value 500 represented by the 5 in the hundreds place. Use a place value chart to show this relationship while writing the equation to reinforce this relationship and repeat with other sets of numbers that have one digit in common such as 3,904 and 5,321.



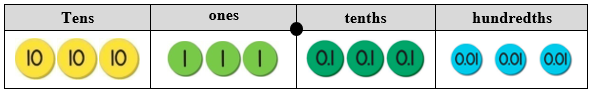
* For example, and . The teacher begins with a ones cube and explains to students that “we are going to model using our base-ten blocks.” Students count out 10 ones cubes and exchange them for a ten rod. The teacher explains that the tens rod represents the value 10, which is 10 times greater than the value 1 represented by the ones cube. Write the equation to reinforce this relationship and repeat this process to model . Then, students exchange a hundreds flat for 10 ten rods to model .The teacher explains that the value represented by a tens rod is 10 times less than the value represented by the hundreds flat and use a place value chart to show this relationship while writing the equation To reinforce this relationship repeat this process to model .



* Instruction includes the use of place value charts and models such as place value disks to demonstrate how the value of a digit changes if the digit moves one place to the left or right. Explicit instruction includes using place value understanding to make the connections between the concepts of “ of,” “ten times less” and “dividing by 10.” Place value charts are used to demonstrate that the decimal point marks the transition between the ones place and the tenths place.
  + For example, students multiply 4 by 10, then record 4 and the product of 40 in a place value chart. This process is repeated by multiplying 40 by 10 while asking students to explain what happens to the digit 4 each time it is multiplied by 10. Next, the teacher explains that multiplying by is the same as dividing by 10. Students multiply 400 by and record the product in their place value chart. This process is repeated, multiplying 40 and 4 by . The teacher asks students to explain how the value of the 4 changed when being multiplied by 10 and .



* + For example, instruction includes using a familiar context such as money, asking students to explain the value of each digit in $33.33. Next, students represent 33.33 in a place value chart using place value disks. Then, students compare the value of the whole numbers (3 dollars and 30 dollars) and compare 0.3 and 0.03 (30 cents and 3 cents). The teacher asks, “How does the value of the three in the hundredths place compare to the value of the three in the tenths place?” and explains that the three in the hundredths place is the value of the three in the tenths place and that multiplying by is the same as dividing by 10.



Instructional Tasks

*Instructional Task 1 (MTR.7.1)*

Paul and his family traveled 528 miles for their summer vacation. Wayne and his family traveled 387 miles for their summer vacation. How much greater is the digit eight in 387 than the digit eight in 528? Have students explain their answer and discuss what role, if any, the other digits play.

*Instructional Task 2 (MTR.7.1)*

At the Sunshine Candy Store, saltwater taffy costs $0.18 per piece.

Part A. How much would 10 pieces of candy cost?

Part B. How much would 100 pieces of candy cost?

Part C. How much would 1000 pieces of candy cost?

Part D. At the same store, you can buy 100 chocolate coins for $89.00. How much does each chocolate coin cost? Explain how you know.

*Instructional Task 3*

Leah wrote the following expressions on her paper:

and

Part A Explain how the value of the 7 in 7.4 changes when it is multiplied by 100. Why does this happen? Part B Explain how the value of the 7 in 7.4 changes when it is multiplied by . Why does this happen?

*Enrichment Task 1 (MTR.5.1)*

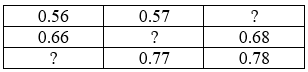
Ana says in 9,396 one 9 has one-tenth the value of the other 9. Is Ana correct? Explain.

*Enrichment Task 2 (MTR.5.1)*

Justin reasoned that in the number 0.444, the value of the 4 in the thousandths place is ten times the value of the 4 in the hundredths place. Is he correct? Explain.

*Enrichment Task 3 (MTR.3.1)*

Students can create decimal grids to represent pattens in decimal place value. Have students start with a decimal that ends in the hundredths. How does the place value of the decimal change when you count forward by hundredths? Backward by hundredths? How does the place value of the decimal change as you add tenths? Hundredths? Have students fill in the blanks to create decimal grid puzzles.



Instructional Items

*Instructional Item 1*

The clues below describe the 4 digits of a mystery number that contains the digits 3,4,7,8.

* The value of the 8 is 10 times the value of the 8 in 3,518.
* The value of the 7 is 100 times the value of the 7 in 1,273.
* The value of the 4 is 100 times the value of the 4 in 7,284.
* The missing place value is the 3.

What is the number?

1. 7,483
2. 8,743
3. 7,834
4. 4,738

*Instructional Item 2*

Which statement correctly compares 0.034 and 34?

1. 0.034 is 10 times the value of 34.
2. 0.034 is the value of 34.
3. 0.034 is the value of 34.
4. 0.034 is the value of 34.

*Instructional Item 3*

0.03 is the value of which number?

1. 0.003
2. 0.3
3. 3
4. 300

*Instructional Item 4*

The clues below describe the 4 digits of a mystery number that contains the following digits:

2, 5, 7, 9.

* The value of the 2 is the value of 2 in 2, 376.
* The value of the 5 is the value of 5 in 549.
* The value of the 7 is the value of 7 in 79,428.
* The value of the 9 is the value of 9 in 9,823.

*Enrichment Item 1 (MTR.3.1)*

0.7 is 100 times as great as which decimal?

a. 0.1

b. 0.7

c. 0.001

d. 0.007

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

### MA.4.NSO.1.5

Benchmark (Grade 4)

MA.4.NSO.1.5 Plot, order and compare decimals up to the hundredths.

*Example:* The numbers 3.2; 3.24 and 3.12 can be arranged in ascending order as 3.12; 3.2 and 3.24.

Benchmark Clarifications:

*Clarification 1:* When comparing numbers, instruction includes using an appropriately scaled number line and using place values of the ones, tenths and hundredths digits.

*Clarification 2:* Within the benchmark, the expectation is to explain the reasoning for the comparison and use symbols (, or ).

*Clarification 3:* Scaled number lines must be provided and can be a representation of any range of numbers.

Benchmark (Grade 5)

MA.5.NSO.1.4 Plot, order and compare multi-digit numbers with decimals up to the thousandths.

*Example:* The numbers 4.891, 4.918 and 4.198 can be arranged in ascending order as 4.198, 4.891 and 4.918.

*Example:*  because is less than , which is the same as .

Benchmark Clarifications:

*Clarification 1:* When comparing numbers, instruction includes using an appropriately scaled number line and using place values of the ones, tenths and hundredths digits.

*Clarification 2:* Within the benchmark, the expectation is to explain the reasoning for the comparison and use symbols (, or ).

*Clarification 3:* Scaled number lines must be provided and can be a representation of any range of numbers.

Connecting Benchmarks/Horizontal Alignment

* MA.4.NSO.2.6, MA.4.NSO.2.7
* MA.4.FR.1.2, MA.4.FR.1.4
* MA.4.DP.1.1, MA.4.DP.1.3
* MA.5.NSO.2.4, MA.5.NSO.2.5
* MA.5.AR.2.1, MA.5.AR.2.2, MA.5.AR.2.3

Terms from the K-12 Glossary

Vertical Alignment

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| --- | --- |
| **Previous Benchmarks**   * MA.3.NSO.1.3 | **Next Benchmarks**  MA. 6.NSO.1.1 |

Purpose and Instructional Strategies

The purpose of this benchmark is for students to plot, order and compare decimals using place value. Grade 4 contains the first work with decimals. During instruction make connections to decimal fractions (e.g., ) (MA.4.FR.1.2).

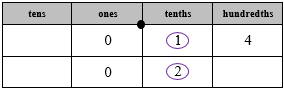
* During instruction, students should apply understanding of flexible representations from MA.4.NSO.1.2 to help them reason while plotting, ordering and comparing.
* For instruction, teachers should show students how to represent these decimals on scaled number lines. Students should use place value understanding to make comparisons.
* Students learn that the names for decimals match their fraction equivalents (e.g., is equivalent to which is equivalent to ).
* Students build area models (e.g., a grid) and other models to compare decimals.
* Instruction expects students to justify their arguments when plotting, comparing and ordering decimals *(MTR.4.1)*.

Common Misconceptions or Errors

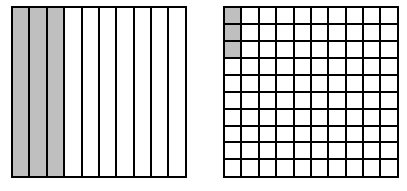
* Students treat decimals as whole numbers when making comparison of two decimals. They think the longer the number, the greater the value. Students may be confused when comparing numbers that have the same number of digits (but different values).
  + For example, they think that 0.04 is greater than 0.4.
* Students may be confused when comparing numbers that have the same digits (but different values).
  + For example, when comparing 2.45 and 13.24, a student may not consider the magnitude of the numbers and only look at their digits. That student may claim that 2.45 is greater than 13.24 because the digit 2 is greater than the digit 1 (though they are actually comparing 2 and 10).
* Students may compare the place values from right to left because many algorithms add and subtract from right to left. Emphasize that the greatest place value is on the left, so they should compare from the greatest place value to the least. Students may find it helpful to model the numbers using base-10 blocks or a place-value chart to visualize decimal place comparisons.

Strategies to Support Tiered Instruction

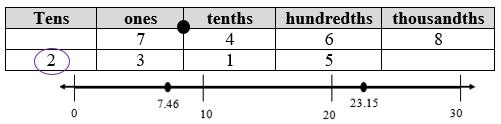
* Instruction includes the use of place value understanding, decimal fractions and decimal grids to compare decimals.
  + For example, students compare 0.14 and 0.2 using decimal fractions. The teacher begins instruction by having students write each decimal as a fraction, and . The teacher explains that is equal to because if we multiply the numerator and denominator of by 10, we generate the equivalent fraction . Next, the teacher compares the fractions to determine that , so .
  + For example, students use place value understanding and a place value chart to compare 0.14 and 0.2. The teacher explains that when comparing decimals, we start with the digit to the far left because we want to compare the greatest place values first. Both values have a 0 in the ones place, so we will move to the tenths place. One tenth is less than two tenths, so .

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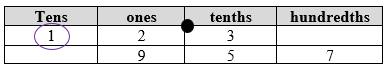
* + For example, students compare 0.3 and 0.03 using decimal fractions. The teacher begins instruction by having students write each decimal as a fraction, and . The teacher then explains to students that is equal to , because if we multiply the numerator and denominator of by 10, we generate the equivalent fraction . Next, the teacher compares the fractions to determine that , so .
  + For example, students compare 0.3 and 0.03 using decimal grids, representing each value and explain that 0.3 covers a greater are of the decimal grid than 0.03, so 0.3 is greater than 0.03.



* Instruction includes the use of place value charts, number lines and relational symbols to compare numbers to the thousandths that have the same amount of digits but different values. It is imperative for students to develop a conceptual understanding of rounding, such as what the benchmarks are, using place value understanding to round numbers without instruction of mnemonics, rhymes or songs.
  + For example, when comparing 7.468 and 23.15, students record 7.468 and 23.15 in a place value chart. The teacher asks students to compare these numbers, beginning with the greatest place value and explains that the number 23.15 has 2 *tens* and the number 7.468 does not have any *tens* so and even though both numbers have the same amount of digits. Also, students plot 7.468 and 23.15 on a number line to compare the magnitude of the numbers.



* + For example, when comparing 12.3 and 9.57 students record 12.3 and 9.57 in a place value chart. The teacher asks students to compare these numbers, beginning with the greatest place value while explaining that the number 12.3 has one *ten* and the number 9.57 does not have any *tens* so and even though both numbers have the same number of digits. Also, students plot 12.3 and 9.57 on a number line to compare the magnitude of the numbers.

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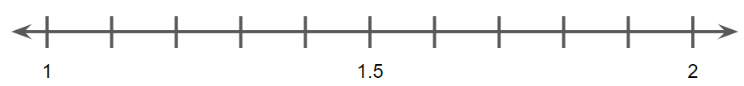
Number line

Instructional Tasks

*Instructional Task 1 (MTR.3.1)*

Use relational symbols to fill in the blanks to compare the numbers.

*Instructional Task 2 (MTR.3.1)*

Part A. Plot the numbers 1.519, 1.9, 1.409 and 1.59 on the number line below.

Part B. Choose two values from the list and compare them using , or .

Part C. Choose a number between 1.519 and 1.59 and plot it on the number line.

Part D. Use evidence from your number line to justify which number is greatest.

*Instructional Task 3 (MTR 3.1)*

List the following numbers in order from least to greatest,

.34, , 23 hundredths, 14 tenths

*Enrichment Task 1 (MTR.6.1)*

Jacob has two different-sized fishbowls. The smaller fishbowl has 1.2 liters of water. The larger fishbowl has 0.92 liter of water. Explain how you can tell whether one fishbowl has more water than the other.

*Enrichment Task 2 (MTR.5.1)*

Use each digit only once to make the comparisons true.

Part A. Use the numbers 3, 6, 9

Part B. Use the numbers 6, 3, 4, and 1.

Instructional Items

*Instructional Item 1*

Select all the values that would make the comparison a true statement.

1. 0.06
2. 0.70
3. 0.8
4. 0.5
5. 0.42

*Instructional Item 2*

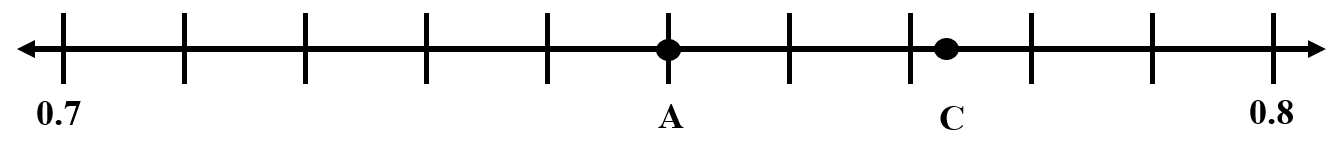
Select all the statements that are true.

*Instructional Item 3*

Order 2.592, 2.59 and 2.399 from least to greatest.

*Instructional Item 4*

Use the number line to complete the questions below.



Part A: What value does Point A represent?

Part B: Which value does Point C represent?

1. 0.77
2. 0.772
3. 0.779
4. 0.78

Part C: Plot Point B between Points A and C. What is the value of Point B?

*Instructional Item 5*

Select the comparison symbol to make the statement true .06 .006 .6

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

**MA.4.NSO.2** *Build an understanding of operations with multi-digit numbers including decimals.*

### MA.4.NSO.2.3 & MA.5.NSO.2.1

Benchmark (Grade 4)

MA.4.NSO.2.3 Multiply two whole numbers, each up to two digits, including using a standard algorithm with procedural fluency.

Benchmark (Grade 5)

MA.5.NSO.2.1 Multiply multi-digit whole numbers including using a standard algorithm with procedural fluency.

Connecting Benchmarks/Horizontal Alignment

* MA.5.FR.2.2
* MA.5.AR.1.1
* MA.5.M.1.1
* MA.5.GR.3.1, MA.5.GR.3.2, MA.5.GR.3.3

Terms from the K-12 Glossary

* Equation
* Expression
* Factor

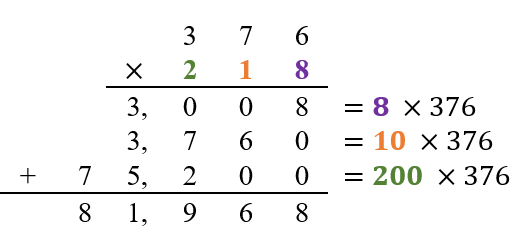
Vertical Alignment

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| --- | --- |
| **Previous Benchmarks**   * MA. 3.NSO.2.2, MA.3.NSO.2.3, MA.3.NSO.2.4 | **Next Benchmarks**   * + MA.6.NSO.2.1 |

Purpose and Instructional Strategies

The purpose of this benchmark is for students in Grade 4 Accelerated is to demonstrate procedural fluency while multiplying multi-digit whole numbers. To demonstrate procedural fluency, students may choose the standard algorithm that works best for them and demonstrates their procedural fluency. A standard algorithm is a method that is efficient, generalizable (it correctly works no matter how many digits are involved) and accurate *(MTR.3.1)*. A standard algorithm is also generalizable meaning that it works no matter how many digits are involved. In the Accelerated Grade 3 course, students had experience multiplying two-digit by three-digit numbers using a method of their choice with procedural reliability (MA.4.NSO.2.2). In Grade 6, students will multiply and divide multi-digit numbers including decimals with fluency (MA.6.NSO.2.1).

* It is important to challenge students to explain the steps they follow when using a standard algorithm (i.e. regrouping, proper recording and placement of digits by place value).
* There is no limit on the number of digits for multiplication in Grade 5.
* When students use a standard algorithm, they should be able to justify why it works conceptually. Teachers can expect students to demonstrate how their algorithm works, for example, by comparing it to another method for multiplication *(MTR.6.1)*.
* Instruction includes helping students understand what each partial product in a standard algorithm represents.
  + For example, when using a standard algorithm to find the product of 376 and 218 students can use their understanding of place value and multiplication to write an equation for each partial product.



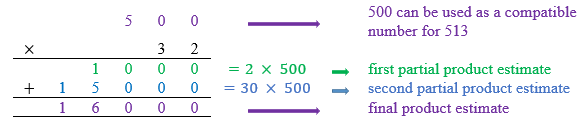
* Along with using a standard algorithm, students should estimate reasonable solutions before solving. Estimation helps students anticipate possible answers and evaluate whether their solutions make sense after solving.
* This benchmark supports students as they solve multi-step real-world problems involving combinations of operations with whole numbers (MA.5.AR.1.1).

Common Misconceptions or Errors

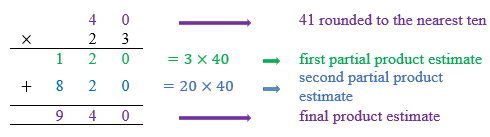
* Students that are taught a standard algorithm without any conceptual understanding will often make mistakes. For students to understand a standard algorithm or any other method, they need to be able to explain the process of the method they chose and why it works. This explanation may include pictures, properties of multiplication, decomposition, etc.
* Some students may struggle with this benchmark if they do not have a strong command of basic addition and multiplication facts.

Strategies to Support Tiered Instruction

* Instruction includes estimating reasonable values for partial products as well as final products.
  + For example, students make reasonable estimates for the partial products and final product for . Before using an algorithm, students can make estimates for partial products and final product to make sure that they are using the algorithm correctly and the answer is reasonable. First, students will estimate the first partial product by rounding 513 to the nearest hundred, 500, and multiplying by 2. When using an algorithm to solve the first partial product, the answer should be approximately 1,000. Next, students can estimate the second partial product by rounding 513 to 500 and multiplying by 30. When using an algorithm to solve the second partial product, it should be approximately 15,000. Finally, students can add the estimates for the partial products to find an estimate for the final product.

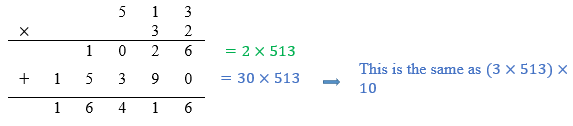
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* + For example, students make reasonable estimates for the partial products and final product for . Before using an algorithm, students can make estimates for our partial products and final product to make sure that they are using the algorithm correctly and the answer is reasonable. First, students will estimate the first partial product by rounding 41 to 40 and multiplying by 3. When using an algorithm to determine the first partial product, it should be approximately 120. Next, students will estimate the second partial product by rounding 41 to 40 and multiplying by 20. When using an algorithm to determine the second partial product, it should be approximately 800. Finally, students can add the estimates for the partial products to find an estimate for the final product.

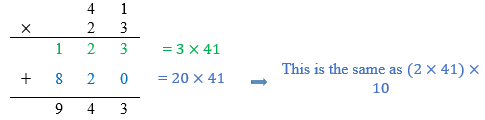
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* Instruction includes explaining and justifying mathematical reasoning while using a multiplication algorithm. Instruction includes determining if an algorithm was used correctly by analyzing any errors made and reviewing the reasonableness of solutions.

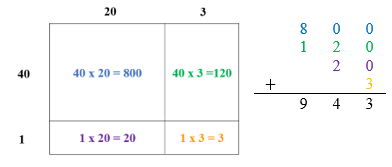
For example, students use an algorithm to determine and explain their thinking using place value understanding. Begin by multiplying 2 ones times 3 ones; students should recognize this equals 6 ones. Students can write the 6 ones under the line, in the ones place. Next, multiply 2 ones times 1 ten, which students should recognize this equals 2 tens. They can write the 2 tens under the line in the tens place. Then, multiply 2 ones times 5 hundreds, which equals 10 hundreds. Write the 10 hundreds under the line in the thousands and hundreds place because 10 hundred is the same as 1 thousand. Students should see that this gives the first partial product of 1,026. Now multiply the 3 ones by the 3 tens from 32; this equals 9 tens or 90. Record 90 below the first partial product of 1,026. Next, multiply the 1 ten by 3 tens, which equal 3 hundreds, and write the 3 in the hundreds place of the second partial product. Then, multiply the 5 hundreds times 3 tens, which equals 15 thousands. Students can write the 15 in the ten thousands and thousands place of our second partial product, noticing that the second partial product is 15,390. Finally, add the partial products to find the product of 16,416.



* + For example, have students use an algorithm to determine and explain their thinking using place value understanding. Explicit instruction could include “Begin by multiplying 3 ones times 1 one. This equals 3 ones. We will write the 3 ones under the line, in the ones place. Next, we will multiply 3 ones times 4 tens. This equals 12 tens. We will write the 12 tens under the line in the hundreds and tens place because 12 tens is the same as 1 hundred 2 tens. This gives us our first partial product of 123. Now we will multiply the 1 one by the 2 tens from 23. This equals 2 tens or 20. We will record 20 below our first partial product of 123. Next, we will multiply 2 tens times 4 tens, which equal 8 hundreds. We will write the 8 in the hundreds place of our second partial product. Our second partial product is 820. Finally, we add our partial products to get 943.”

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* + For example, students solve using an area model and place value understanding and explain how each partial product is calculated and what it represents as they multiply using the area model. Then, students explain how the final product is calculated using the partial products from the area model.



Instructional Tasks

*Instructional Task 1 (MTR.4.1, MTR.5.1)*

Using the digits 1, 2, 3 and 4, arrange them to create two 2-digit numbers that when multiplied, will yield the greatest product.

*Instructional Task 2 (MTR.7.1)*

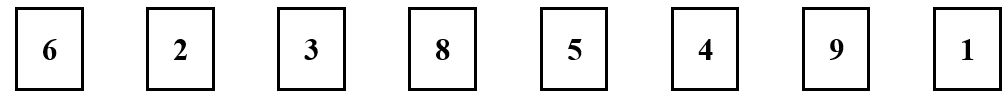
Maggie has three dogs. She buys a box containing 175 bags of dog food. Each bag weighs 64 ounces.

Part A. What is the total weight of the bags of dog food in ounces?

Part B. Maggie has a storage cart to transport the box that holds up to 750 pounds. Will the storage cart be able to hold the box? Explain.

*Instructional Task 3*

Select six cards from the set of cards below.



Part A. Use your cards to create a four-digit number and a two-digit number. Find the product of your four-digit and two-digit numbers.

Part B. Use your cards to create a three-digit number and another three-digit number. Find the product of your three-digit numbers.

Part C. How are your two products similar? How are they different?

*Instructional Task 4 (MTR.4.1, MTR.5.1)*

Using the digits 2, 3, 6, and 8 arrange them to create two 2-digit numbers that when multiplied, will yield the smallest product.

Instructional Items

*Instructional Item 1*

Select the expressions that have a product of 480.

*Instructional Item 2*

What is the product of 427 ?

*Instructional Item 3*

Select the expression that has a product of 255.

1. 15 X 17
2. 25 X 13
3. 25 X 25
4. 75 X 15

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

### MA.4.NSO.2.4 & 5.NSO.2.2

Benchmark (Grade 4)

MA.4.NSO.2.4 Divide a whole number up to four digits by a one-digit whole number with procedural reliability. Represent remainders as fractional parts of the divisor.

Benchmark Clarifications:

*Clarification 1:* Instruction focuses on helping a student choose a method they can use reliably.

*Clarification 2:* Instruction includes the use of models based on place value, properties of operations or the relationship between multiplication and division.

Benchmark (Grade 5)

MA.5.NSO.2.2 Divide multi-digit whole numbers, up to five digits by two digits, including using a standard algorithm with procedural fluency. Represent remainders as fractions.

*Example:* The quotient gives with remainder which can be expressed as .

Benchmark Clarifications:

*Clarification 1:* Within this benchmark, the expectation is not to use simplest form for fractions.

Connecting Benchmarks/Horizontal Alignment

* MA.5.FR.2.4
* MA.5.AR.1.1, MA.5.AR.1.3
* MA.5.M.1.1
* MA.5.GR.3.3

Terms from the K-12 Glossary

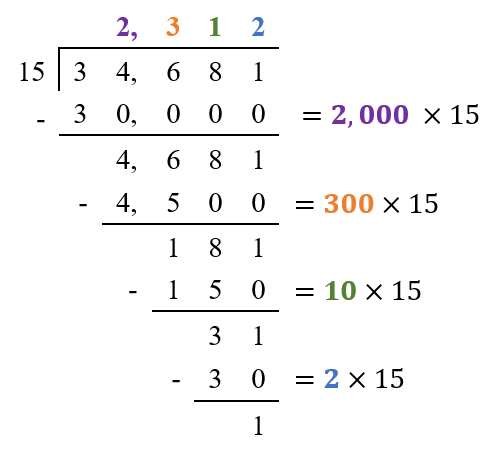
* Dividend
* Divisor
* Expression
* Equation
* Quotient
* Whole Number

Vertical Alignment

|  |  |
| --- | --- |
| **Previous Benchmarks**   * MA.3.NSO.2.4 | **Next Benchmarks**   * MA.6.NSO.2.1 |

Purpose and Instructional Strategies

The purpose of this benchmark is for students to demonstrate procedural fluency while dividing multi-digit whole numbers with up to 5-digit dividends and 2-digit divisors. To demonstrate procedural fluency, students in Grade 4 Accelerated may choose a standard algorithm that works best for them and demonstrates their procedural fluency. A standard algorithm is a method that is efficient, generalizable (it works correctly no matter how many digits are involved) and accurate *(MTR.3.1)* no matter how many digits are involved. In Grade 6, students will multiply and divide multi-digit numbers including decimals with fluency (MA.6.NSO.2.1).

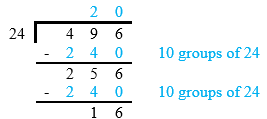
* When students use a standard algorithm, they should be able to justify why it works conceptually. Teachers can expect students to demonstrate how their algorithm works, for example, by comparing it to another method for division *(MTR.6.1)*.
* When used efficiently, partial quotients are a suitable procedure for dividing multi-digit numbers. It can often be more reliable and efficient for students than the long division algorithm. Students can demonstrate fluency by skillfully using partial quotients and should be able to understand the full long division algorithm.
  + For example, students can use place value understanding, estimation and partial quotients to solve Students can begin by determining that the product gets them close to their dividend of 34,681 and continue by subtracting groups of 15 based place value and multiples of 15.
* In this benchmark, students are to represent remainders as fractions. In the benchmark example, the quotient of is represented as . Students should gain understanding that this quotient means that there are 3 full groups of 7 in 27, and the remainder of 6 represents of another group. Students are not expected to have mastery of converting between forms (fraction, decimal, percentage) until grade 6 but students should start to gain familiarity that fractions and decimals are numbers and can be equivalent (i.e., a remainder of is the same as 0.5). Writing remainders as fractions or decimals is acceptable. Similarly, students should be able to understand that a remainder of zero means that whole groups have been filled without any of the dividend remaining *(MTR.5.1, MTR.7.1)*.
* Along with using a standard algorithm, students should estimate reasonable solutions before solving. Estimation helps students anticipate possible answers and evaluate whether their solutions make sense after solving.
* When working with standard algorithms, it is important to note that when used well, partial quotients is a suitable procedure, even though it is technically not an “algorithm.” Partial quotients is not considered an algorithm because the steps are not fully prescribed; they involve some choice by the student. This procedure is often more reliable and efficient for a student than the long division algorithm that is considered “standard” in the U.S. Students can demonstrate fluency by skillfully using partial quotients, and any such student should be able to understand the full long division algorithm.
* This benchmark supports students as they solve multi-step real-world problems involving combinations of operations with whole numbers (MA.5.AR.1.1). In a real-world problem, students should interpret remainders depending on its context.

Common Misconceptions or Errors

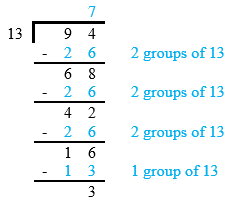
* Students can make computational errors while using standard algorithms when they cannot reason why their algorithms work. In addition, they can struggle to determine where or why that computational mistake occurred because they did not estimate reasonable values for intermediate outcomes as well as for the final outcome. During instruction, teachers should expect students to justify their work while using their chosen algorithms and engage in error analysis activities to connect their understanding to the algorithm.

Strategies to Support Tiered Instruction

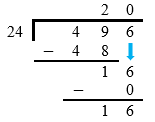
* Instruction includes estimating reasonable values for quotients when dividing by two-digit divisors.
  + For example, students make reasonable estimates for the quotient of . Before using an algorithm, students can estimate the quotient to make sure that they are using the algorithm correctly and the answer is reasonable. Students can use multiples of 24 and their understanding of multiplication and division to estimate the quotient. Students may want their estimate to be as close to 496 as possible. So, knowing that , they can state that . A reasonable estimate for the quotient would be 20 because 480 is close to 496.”
  + For example, students make reasonable estimates for the quotient of . Explicit instruction could include stating, “Before using an algorithm, we will estimate the quotient to make sure that we are using the algorithm correctly and our answer is reasonable. The divisor of 13 is close to 10 and the dividend of 94 is close to 90. So, we can use to estimate that our quotient should be close to 9.”
* Instruction includes explaining and justifying mathematical reasoning while using a division algorithm to divide by two-digit divisors. Instruction also includes determining if an algorithm was used correctly by analyzing any errors made and reviewing the reasonableness of solutions.
  + For example, the teacher connects place value with the partial quotients model to determine . Students should not just view the digits as individual numbers but connect individual digits with the value of that number (e.g., 496 is ). Instruction includes stating, “In this problem we are finding how many groups of 24 are in 496. We will subtract groups of 24 until we cannot subtract any more groups. The total number of groups that we can subtract is the quotient. We can subtract 10 groups of 24 two times, so the quotient is 20. We have a remainder of 16. The quotient is represented as 20 because we have 20 full groups of 24 in 496 and the remainder of 16 represents of another group.”



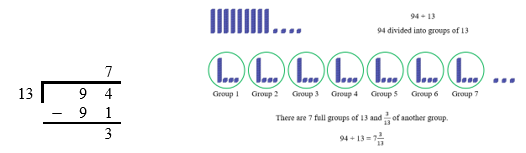
* + For example, connect place value with the partial quotients model to determine . Students should not just view the digits as individual numbers but connect individual digits with the value of that number (e.g., 94 is ). Instruction includes stating, “In this problem we are finding how many groups of 13 are in 94. We will subtract groups of 13 until we cannot subtract any more groups. The total number of groups that we can subtract is the quotient. We can subtract 7 groups of 13, so the quotient is 7. We have a remainder of 3. The quotient is represented as 7 because we have 7 full groups of 13 in 94 and the remainder of 3 represents of another group.”



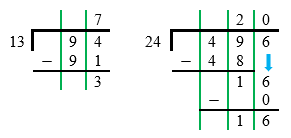
* + For example, students use an algorithm to solve and explain their thinking using place value understanding. Instruction includes stating, “In this problem we are finding how many groups of 24 are in 496. We will begin by dividing our largest place value first. Recognizing that the 4 represents 400, if your divide 400 by 24 the result will be less than 100, so the quotient won’t have any whole hundreds. Remember that 496 is the same as , so we will see how many groups of 24 are in . We can also think of this as . There are 20 groups of 24 in , that’s 2 times 10 groups, so we can place a 2 in the tens place of the quotient. Next, we will subtract (20 groups of 24 equal ) to find a difference of . We can combine this with the remaining in 496. We now have remaining from our original dividend of 496, this is not enough to make a group of 24. We have a remainder of 16. The quotient is represented as because we have 20 full groups of 24 in 496 and the remainder of 16 represents of another group. Our quotient of is close to our estimate of 20, this helps us determine that our answer is reasonable.”



* + For example, students use an algorithm to solve and explain their thinking using base ten blocks and place value understanding. Instruction includes stating, “In this problem we are finding how many groups of 13 are in 94. We will begin by dividing our largest place value first. How many groups of 13 are in ? Recognizing that the 9 represents 90, if you divide 90 by 13 the result will be less than 10, so the quotient won’t have any whole tens. Remember that 94 is the same as and , so we will see how many groups of 13 are in . We can also think of this as. There are 7 groups of 13 in . Next, we will subtract to find our remainder of 3. Our quotient is represented as because we have 7 full groups of 13 in 94 and the remainder of 3 represents of another group. Our quotient of is close to our estimate of 9, this helps us determine that our answer is reasonable.”

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* Instruction includes the use of place value columns to support place value understanding when using a division algorithm.
  + Example:

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Instructional Tasks

*Instructional Task 1 (MTR.5.1)*

The Magnolia Outreach organization is donating 6,924 pounds of rice to families in need. They pour all the rice into 15-pound containers.

Part A. How many containers will they fill completely if they use all the rice?

Part B. How many containers will they need for all of the rice?

Part C. Will the Magnolia Outreach organization completely fill all of the containers needed for all of the rice? If not, will the partially filled container be more or less than half-full? Explain how you know.

*Instructional Task 2*

Natalie says that 5,978 is not divisible by 24 a whole number of times. Is Natalie correct? Why or why not?

Instructional Items

*Instructional Item 1*

What is the quotient of ?

*Instructional Item 2*

Part A. What is the quotient of 65?

Part B. Write the remainder of 65, as a fraction.

*Instructional Item 3*

What is the quotient of ?

1. 20
2. 21
3. 201
4. 201

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

### MA.4.NSO.2.6

Benchmark

MA.4.NSO.2.6 Identify the number that is one-tenth more, one-tenth less, one-hundredth more and one-hundredth less than a given number.

*Example:* One-hundredth less than 1.10 is 1.09.

*Example:* One-tenth more than 2.31 is 2.41.

Connecting Benchmarks/Horizontal Alignment

* MA.4.NSO.1.5
* MA.4.FR1.1, MA.4.FR.1.2
* MA.4.M.1.2
* MA.4.M.2.2

Terms from the K-12 Glossary

* Equation
* Expression

Vertical Alignment

|  |  |
| --- | --- |
| **Previous Benchmarks**   * MA.2.NSO.2.2 | **Next Benchmarks**   * MA.5.NSO.2.3, MA.5.NSO.2.4 |

Purpose and Instructional Strategies

The purpose of this benchmark is for students to develop an understanding of place value with tenths and hundredths in addition and subtraction.

* This benchmark extends upon students’ thinking about 1 more/less from whole numbers to decimals. Students should continue using place value understanding to reason how adding and subtracting 1 tenth and 1 hundredth changes a number’s value.
* Teachers should use familiar manipulatives to help connect students’ exploration of decimals to whole numbers. These materials include place-value blocks, tenths and hundredths charts (modeled after hundred charts students used in primary), and place value mats. During instruction, teachers model correct vocabulary consistently to describe decimals and expect the same from students (e.g., the number 1.09 is read as “one *and* 9 hundredths”).
* In this initial exploration of decimal addition and subtraction, the expectation is to develop understanding using manipulatives, visual models, discussions, estimation and drawings, with the focus being on adding and subtracting 1 tenth and 1 hundredth. This prepares students for the broader exploration of adding and subtracting decimals in MA.4.NSO.2.7.

Common Misconceptions or Errors

* When using place value blocks, it is important to first identify the value of each block. Students may have preconceptions about relating units to ones, rods to tens, and flats to hundreds, which can be confusing when their values shift from whole numbers to decimals. Teachers should share the relationship between the blocks (each larger block is ten times larger than the next smaller block) so that students understand they can be used flexibly.
* Students may struggle to understand that one-hundredth is smaller than one-tenth because of one hundred is greater than one ten. During instruction, emphasize that one-hundredth is less because it would require 100 hundredths to equal 1 whole and only 10 tenths to equal 1 whole.
* Students may not understand the reasoning behind the placement of the decimal point. Modeling and exploring the relationships between place-values will help students gain understanding.

Strategies to Support Tiered Instruction

* Instruction includes opportunities to model and represent decimals.
  + For example, if a 10 by 10 grid of 100 represents one whole, students shade in 0.4 on the grid using the appropriate language to connect “four-tenths” to the decimal 0.4. Then, students shade in what 0.12 represents. The teacher connects the language “twelve hundredths” to the decimal 0.12. Students compare the decimals using visuals. This will help solidify the understanding that tenths are larger than hundredths. Using visuals will also connect the learning of one tenth more/less and one hundredth more/less.

|  |  |
| --- | --- |
| **Decimal grid.** | **Decimal grid.** |
| 0.4 | 0.12 |

* Instruction includes building decimals with place value blocks.
* For example, the teacher asks students to build 0.3 (three-tenths) and 0.4 (four-tenths).

|  |  |
| --- | --- |
| **Three tens rods.** | **Four tens rods.** |
| 0.3 | 0.4 |

Students physically see that 0.3 is one-tenth less than 0.4.

* During instruction, the teacher shares the relationship between the blocks (each larger block is ten times larger the next smaller block) to demonstrate that they can be used flexibly.
* For example, emphasize that one-hundredth is smaller because it would require 100 hundredths to equal 1 whole and only 10 tenths to equal 1 whole.

Instructional Tasks

*Instructional Task 1 (MTR.4.1)*

Vianna says that 1 tenth more than 3.9 is 4. Tracy says that 1 tenth more than 3.9 is 3.91. Who is correct? Explain how you know.

*Instructional Task 2 (MTR.4.1)*

Robert says that 5.21 is equivalent to 5.210. James says that they are not equivalent. Who is correct? Explain how you know.

*Enrichment Task 1 (MTR.1.1)*

Molly says, “0.30 is greater than 0.3 because 30 is greater than 3.” Do you agree? Explain.

*Enrichment Task 2 (MTR.7.1)*

Rachel runs the 100-meter race in 11.13 seconds. Courtney ran the same race in 11.12 seconds. Who had the faster time? How do you know?

Instructional Items

*Instructional Item 2*

What is one tenth more than 3.8?

What is one tenth less than 7?

What is one hundredth more than 15.29?

What is one hundredth less than 7?

*Instructional Item 2*

*What is the equivalent decimal to 4.2?*

*What is one tenth less than 3?*

*What is one hundredth more than 4.23?*

*What is one hundredth less than 2.80?*

*Enrichment Item 1 (MTR.6.1)*

Will the product of be greater than, less than, or the same as 73.8? Explain how you know.

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

### MA.4.NSO.2.7 & MA.5.NSO.2.3

Benchmark (Grade 4)

MA.4.NSO.2.7 Explore the addition and subtraction of multi-digit numbers with decimals to the hundredths.

Benchmark Clarifications:

*Clarification 1:* Instruction includes the connection to money and the use of manipulatives and models based on place value.

Benchmark (Grade 5)

MA.5.NSO.2.3 Add and subtract multi-digit numbers with decimals to the thousandths, including using a standard algorithm with procedural fluency.

Connecting Benchmarks/Horizontal Alignment

* MA.4.NSO.1.5
* MA.4.FR.1.1, MA.4.FR.1.2
* MA.4.FR.2.3
* MA.4.M.2.2
* MA.4.DP.1.3
* MA.5.NSO.1.5
* MA.5.AR.2.1, MA.5.AR.2.2, MA.5.AR.2.3
* MA.5.M.2.1
* MA.5.GR.2.1

Terms from the K-12 Glossary

* Equation
* Expression

Vertical Alignment

|  |  |
| --- | --- |
| **Previous Benchmarks**   * MA.2.NSO.2.2 * MA.4.NSO.2.6 | **Next Benchmarks**   * MA.6.NSO.2.3 |

Purpose and Instructional Strategies

The purpose of this benchmark is for students to explore addition and subtraction of decimals to the thousandths using manipulatives, visual models, discussions, estimation and drawing.

* Instruction should focus on strategies based on place value. Through the connection to money, students can build on previous content knowledge about money to adding and subtracting decimals based on place value. Examples of manipulatives that support understanding when adding and subtracting decimals are base-ten blocks, place value chips, money (dollars and coins) and place value mats.
* To demonstrate procedural fluency, students may choose a standard algorithm that works best for them and demonstrates their procedural fluency. A standard algorithm is a method that is efficient, generalizable (it works correctly no matter how many digits are involved) and accurate *(MTR.3.1)*. A standard algorithm is also generalizable meaning that it works no matter how many digits are involved.
* Instruction includes explaining that lining up the decimals points to add or subtract decimals also lines up the corresponding place values. Due to the fixed denominator associated with each place value, such as or , the process of adding or subtracting the digits in a specific column is the same as adding or subtracting fractions with like denominators.
* When students use a standard algorithm, they should be able to justify why it works conceptually. Teachers can expect students to demonstrate how their algorithm works, for example, by comparing it to another method for addition and subtraction *(MTR.6.1)*.
* Instruction makes the connection to decimal and fraction equivalence when solving addition and subtraction equations involving different numbers of decimal places.
  + For example, when solving students can use their understanding of decimal and fraction equivalence to write 7.5 as 7.50 because *seven and five tenths* is equivalent to *seven and fifty hundredths.*
* Along with using a standard algorithm, students should estimate reasonable solutions before solving. Estimation helps students anticipate possible answers and evaluate whether their solutions make sense after solving.

Common Misconceptions or Errors

* A common error that students make is to not add or subtract like place values, especially in an example such as . Instruction should relate decimals to methods used for whole numbers. When adding whole numbers, ones were added to ones, tens to tens, hundreds to hundreds, and so forth. When adding decimal numbers, like place values are combined, too. Like place values are subtracted, just as with whole numbers.
* Students can make computational errors while using standard algorithms when they cannot reason why their algorithms work. In addition, they can struggle to determine where or why that computational mistake occurred because they did not estimate reasonable values for intermediate outcomes as well as for the final outcome. During instruction, teachers should expect students to justify their work while using their chosen algorithms and engage in error analysis activities to connect their understanding to the algorithm.

Strategies to Support Tiered Instruction

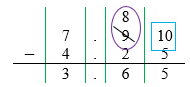
* Instruction includes estimating reasonable values for sums and differences when adding and subtracting decimals to the hundredths.
  + For example, students make reasonable estimates for the sum of . Instruction includes stating, “Before using an algorithm, we will estimate the sum to make sure that we are using the algorithm correctly and our answer is reasonable. I will use my understanding of rounding decimals to estimate my sum. The addend of 6.32 rounds to 6 when rounded to the nearest whole number and the addend 2.84 rounds to 3 when rounded to the nearest whole number. A reasonable estimate for my sum would be 9 because ”
  + For example, students make reasonable estimates for the difference of . Instruction includes stating, “Before using an algorithm, we will estimate the difference to make sure that we are using the algorithm correctly and our answer is reasonable. I will use my understanding of rounding decimals to estimate my difference. The minuend of 7.9 rounds to 8 when rounded to the nearest whole number and the subtrahend 4.25 rounds to 4 when rounded to the nearest whole number. A reasonable estimate for my difference would be 4 because ”
* Instruction includes explaining and justifying mathematical reasoning while using an algorithm to add and subtract decimals to the hundredths. Instruction also includes determining if an algorithm was used correctly by analyzing any errors made and reviewing the reasonableness of solutions.
  + For example, students use a standard algorithm to determine and explain their thinking using a place value understanding. Instruction includes stating, “Begin by lining up the decimal points and place values for each addend

Next, add in hundredths place. . Because the total number of is less than it is not necessary to regroup. Next, add in the tenths place. . Because I have more than it

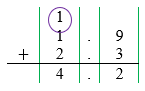
is necessary to regroup the to make one whole. After composing a group of 10 tenths there is 1 tenth remaining. Finally, add plus and the 1 whole that was regrouped from the tenths place. The sum is 9.16. Our sum of 9.16 is close to our estimate of 9, this helps us determine that our answer is reasonable.”



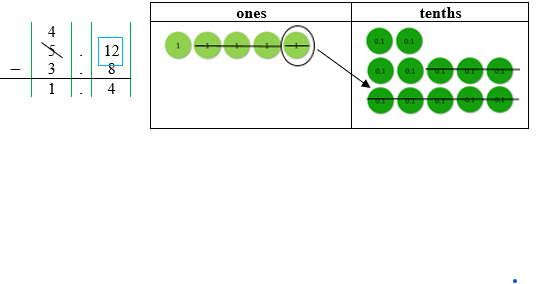
* + For example, students use a standard algorithm to determine and explain their thinking using place value understanding. The teacher reminds students that 7.9 is equivalent to 7.90 and uses a decimal grid to show the equivalency of 0.9 and 0.90 if needed. Instruction includes stating, “Begin by lining up the decimal points and place values. Next, subtract 4.25 starting in the place. There are not enough to subtract from . It is necessary to decompose one tenth into . Now there are , and there is enough to subtract . . Then, subtract the : . Finally, subtract the : . The difference is 3.65. Our difference of 3.65 is close to our estimate of 4, this helps us determine that our answer is reasonable.”



* + For example, students use a standard algorithm to determine and explain their thinking using a place value understanding. Instruction includes stating, “Begin by lining up the decimal points and place values for each addend. Next, add in place. . Because I have more than it is necessary to regroup the to make one whole. After composing a group of there are remaining. Finally, add plus and the 1 whole that was regrouped from the place. The sum is 4.2. Our sum of 4.2 is close to our estimate of 4, this helps us determine that our answer is reasonable.”



* + For example, students use a standard algorithm to determine and explain their thinking using place value disks and their understanding of place value. Instruction includes stating, “Begin by lining up the decimal points and place values. Next, subtract 3.8 starting in the place. There are not enough to subtract from . It is necessary to decompose one whole into . Now there are a total of , and there are enough to subtract . . Finally, subtract the : . The difference is 1.4. Our difference of 1.4 is close to our estimate of 1, this helps us determine that our answer is reasonable.”



* + Instruction includes the use of place value columns to support place value understanding when using an algorithm to add and subtract decimals

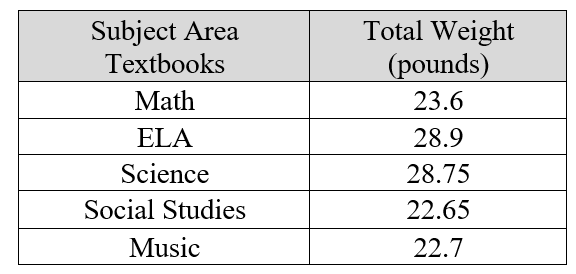
Instructional Tasks

*Instructional Task 1*

Karina’s lunchbox weighs 2.5 pounds. She took out her apple that weighed 0.65 pounds. How much does her lunchbox weigh now?

*Instructional Task 2*

A school is packing textbooks into a shipping container. The table below shows the total weights of the textbooks for five different subject areas.



If the weight limit of the shipping container is 75 pounds, which combination of subject area textbooks will fit in the shipping container?

1. Math, Science and Music
2. ELA, Science and Social Studies
3. Math, ELA and Science
4. Math, Science and Social Studies

*Instructional Task 3 (MTR.7.1)*

Anna’s backpack weighs 10.7 pounds. She added her math book to her backpack and it now weighs 17.3 pounds. How does her math book weigh?

*Enrichment Task 1 (MTR.7.1)*

The average rainfall, in inches, for five cities is shown.

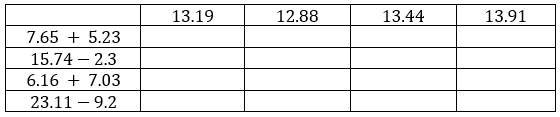


What is the difference between the greatest and the least amount of rainfall?

Instructional Items

*Instructional Item 1*

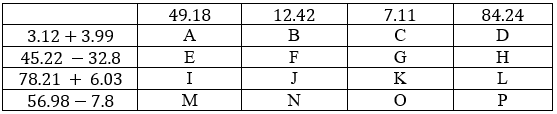
Match each expression on the left with the equivalent decimal.



*Instructional Item 2*

An equation is shown. What is the missing value in the equation?

*Instructional Item 3*

**

*Enrichment Item 1 (MTR.3.1)*

Select all that are equivalent to 1.75.

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

**MA.5.NSO.1** *Understand the place value of multi-digit numbers with decimals to the thousandths place.*

### MA.5.NSO.1.2

Benchmark

MA.5.NSO.1.2 Read and write multi-digit numbers with decimals to the thousandths using standard form, word form and expanded form.

*Example:* The number sixty-seven and three hundredths written in standard form is and in expanded form is or .

Connecting Benchmarks/Horizontal Alignment

* MA.5.NSO.2.4, MA.5.NSO.2.5
* MA.5.AR.2.1, MA.5.AR.2.2, MA.5.AR.2.3
* MA.5.M.2.1

Terms from the K-12 Glossary

Vertical Alignment

|  |  |
| --- | --- |
| **Previous Benchmarks**   * MA.4.NSO.1.2 | **Next Benchmarks**   * + MA.6.AR.1.1 |

Purpose and Instructional Strategies

The purpose of this benchmark is for students to read numbers appropriately and to write numbers in all forms. Utilizing place value, students are expected to understand the value of tenths, hundredths, and thousandths, extending from their work to read and write whole numbers in any form in the Grade 3 Accelerated course (MA.4.NSO.1.2). Writing numbers in expanded form can help students see the relationship between decimals and fractions *(MTR.5.1)*.

Translating from written form to symbolic form builds the foundation for moving from written to algebraic form in grade 6 (MA.6.AR.1.1).

* Representing numbers in flexible ways will help students name, order, compare and operate with decimals (MTR.3.1).
* During instruction, teachers should relate all three forms using place value charts and base ten manipulatives (e.g., blocks) (MTR.3.1, MTR.4.1, MTR.5.1).

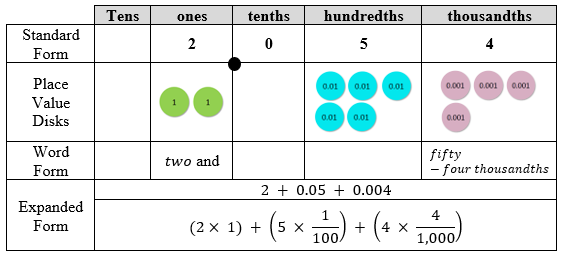
Instruction includes explanation of the word *and* when reading decimals and writing decimals in word form. Students should understand that the word *and* represents the decimal point in the number and separates the portions of the number that are greater than and less than 1

Common Misconceptions or Errors

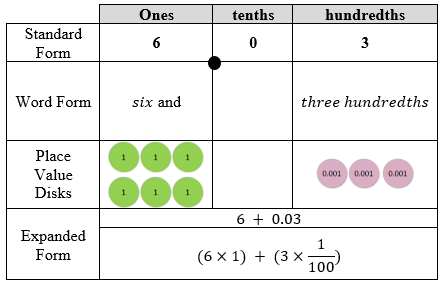
* Students may incorrectly read and write from expanded form if one of the digits is 0, like in the number 67.03 as used in the benchmark example. A common mistake that students make is to name the number as 67.3 because they do not see that 3 is the value of hundredths.

Strategies to Support Tiered Instruction

* Instruction includes the use of place value understanding, models such as place value disks and decimal fractions to read and write multi-digit numbers with decimals to the thousandths using standard form, word form and expanded form when one of the digits in the decimal place values is 0.
  + For example, write 2.054 in standard form, word form and expanded form using a place value chart.

****

* + - For example, the teacher uses decimal fractions and a place value chart to help students read 2.054, modeling how to write the decimal portion of the number as a fraction, and explaining that doing so helps us to read the decimal correctly. Also, the teacher explains that the word “and” is used for a portion of a number, decimal or fraction. Next, the teacher and students write 2.054 as and read the number as “two and fifty-four thousandths.”
    - For example, write 6.03 in standard form, word form and expanded form using a place value chart.



* + For example, the teacher uses decimal fractions and a place value chart to help students read 6.03, while modeling how to write the decimal portion of the number as a fraction, and explaining that doing so helps us to read the decimal correctly. Also, the teacher explains that the word “and” is used for a portion of a number, decimal or fraction. Next, write 6.03 as and read the number as “six and three hundredths.”
* Opportunities for enrichment include presenting problems with decimals in different notations.
  + For example, students can be asked to solve the problem Thirty-one and fifty-three tens minus (2 x10) + (5x1) + (4x 1/10) + (9x 1/100). Students would need to identify the numbers in word form and expanded form, rewrite the numbers in standard form and solve.

Instructional Tasks

*Instructional Task 1*

Use the number cards below to write a number in standard, word and expanded forms. You can use the cards in any order to make your number, but it must have a digit other than zero in the thousandths place.

.

8

0

3

6

5

*Instructional Task 2*

Micah says that the expanded form of 3.627 is

Jayden says that the expanded form of 3.627 is .

Their teacher explains that they are both correct. Explain why this is the case.

*Enrichment Task 1 (MTR.5.1)*

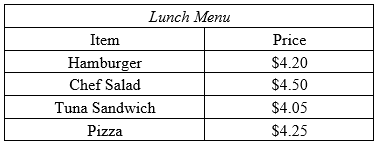
Sherri biked 24.008 kilometers. What is 24.008 written in expanded form?

(2 x \_\_\_\_\_\_) + (4 x \_\_\_\_\_\_\_\_) + (8 x \_\_\_\_\_\_\_\_\_)

*Enrichment Task 2 (MTR.3.1)*

Matthew is building a fence. He has a piece of wood that measures (9x 1/10) + (6x 1/100) + (4x 1/1000) meter. How can this measurement be written as a decimal?

*Enrichment Task 3 (MTR.7.1)*



Melanie spent eight dollars and seventy-five cents on lunch. What two items did Melanie buy?

Instructional Items

*Instructional Item 1*

Which shows the number below in word form?

*Instructional Item 2*

Write in standard form.

*Instructional Item 3*

Which of the following correctly represents 9.023? Select all that apply.

1. *Nine and twenty-three hundredths*
2. *Nine and twenty-three thousandths*

*Enrichment Item 1 (MTR.3.1)*

Which two decimals are equivalent to (8x100) + (3x 1/10) + (6x 1/100)?

1. 8.36
2. 800.36
3. 800.036
4. 800.306
5. 8.360
6. 800.360

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

### MA.5.NSO.1.3

Benchmark

MA.5.NSO.1.3 Compose and decompose multi-digit numbers with decimals to the thousandths in multiple ways using the values of the digits in each place. Demonstrate the compositions or decompositions using objects, drawings and expressions or equations.

*Example:* The number can be expressed as or as .

Connecting Benchmarks/Horizontal Alignment

* MA.5.NSO.2.4/2.5
* MA.5.AR.2.1/2.2/2.3
* MA.5.M.2.1

Terms from the K-12 Glossary

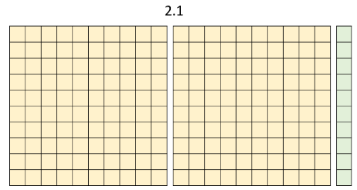
Vertical Alignment

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| --- | --- |
| **Previous Benchmarks**   * MA. 4.FR.2.1 | **Next Benchmarks**   * + MA.6.NSO.3.2 |

Purpose and Instructional Strategies

The purpose of this benchmark is for students to use place value relationships to compose and decompose multi-digit numbers with decimals. While students have composed and decomposed whole numbers (MA.3.NSO.1.2) and fractions (MA.4.FR.2.1) in the Grade 3 Accelerated course, naming multi-digit decimals in flexible ways in grade 5 helps students with decimal comparisons and operations (addition, subtraction, multiplication and division). Flexible representations of multi-digit numbers with decimals also reinforces the understanding of how the value of digits change if they move one or more places left or right (MA.5.NSO.1.1). Composing and decomposing numbers also helps build the foundation for further work with the Distributive property in Grade 6 (MA.6.NSO.3.2).

* Instruction may include multiple representations using base ten models *(MTR.2.1)*. During instruction, teachers should emphasize that the value of a base ten block (or another concrete model) is flexible (e.g., one flat could be 1 ten, one, tenth, hundredth, and so forth). Using base ten models flexibly helps students think about how numbers can be composed and decomposed in different ways.
  + For example, the image below shows 2.1. This representation shows that 2.1 can also be composed as or . Thinking about 2.1 as may help subtracting easier for students because they can think about the expression as , or .
* Representing multi-digit numbers with decimals flexibly can help students reason through multiplication and division as well. For example, students may prefer to multiply as to use more familiar numbers *(MTR.2.1, MTR.5.1)*.
* Students should name their representations in different forms (e.g., word, expanded) during classroom discussion. While students are representing multi-digit numbers with decimals in different ways, teachers should invite all answers and have students compare them *(MTR.4.1).*

Common Misconceptions or Errors

* Students may assume that the value of base ten blocks are fixed based on their previous experiences with whole numbers (e.g., units are ones, rods are tens, flats are hundreds). During instruction, teachers should name a base ten block for each example so students can relate the other blocks. (For example, “Show 2.4. Allow 1 rod to represent 1 tenth.”)

Strategies to Support Tiered Instruction

|  |
| --- |
| * Instruction includes opportunities to decompose multi-digit numbers with decimals to the hundredths in multiple ways. Instruction includes the use of base-ten blocks to represent decimals where one flat represents one whole, one rod represents one tenth and one unit represents one hundredth. During instruction, the teacher names a base ten block for each example, so students relate the other blocks. A chart can be used to organize students’ thinking. The teacher asks students to identify the different ways to name the values (grouping the hundredths into tenths and the tenths into the ones, e.g., 2 *ones* and 34 *hundredths* or 20 *tenths* and 34 *hundredths*, etc.)   + For example, decompose 2.34 in multiple ways using ones, tenths and hundredths.   **Chart showing decomposition of numbers.** |
| * + For example, show 3.5. Allow one rod to represent one-tenth. Then, decompose 3.5 in multiple ways using ones, tenths and hundredths.   Multiple examples of representation of numbers. |

Instructional Tasks

*Instructional Task 1 (MTR.4.1, MTR.5.1)*

Using base ten blocks, represent 1.36 in two different ways. Allow one flat to represent 1 whole.

*Instructional Task 2 (MTR.3.1)*

How many tenths are equivalent to 13.2? How do you know?

*Enrichment Task 1 (MTR.7.1)*

The queen honey bee is the largest bee in her colony. A scientist measures one queen honey bee’s length to be 3.024 centimeters. Write this decimal in word form and tell the place and value of the digit 4.

*Enrichment Task 2 (MTR.6.1)*

Nicole says that 6.493 can be written as 6 ones + 4 tenths + 9 hundredths + 3 thousandths and as 6 ones + 493 hundredths. Do you agree.

Instructional Items

*Instructional Item 1*

Select the expressions that have a product of 480.

*Instructional Item 2*

What is the product of 427 ?

*Instructional Item 3*

Select the expression that has a product of 255.

1. 15 X 17
2. 25 X 13
3. 25 X 25
4. 75 X 15

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

### MA.5.NSO.1.4

### See Benchmark MA.4.NSO.1.5

### MA.5.NSO.1.5

Benchmark

MA.5.NSO.1.5 Round multi-digit numbers with decimals to the thousandths to the nearest hundredth, tenth or whole number.

*Example:* The number 18.507 rounded to the nearest tenth is 18.5 and to the nearest hundredth is 18.51.

Connecting Benchmarks/Horizontal Alignment

* MA.5.NSO.2.3/2.4
* MA.5.AR.2.1/2.2/2.3

Terms from the K-12 Glossary

Vertical Alignment

|  |  |
| --- | --- |
| **Previous Benchmarks**   * MA.4.NSO.1.4 | **Next Benchmarks**   * MA.6.NSO.2.3 * MA.8.NSO.1.4 |

Purpose and Instructional Strategies

The purpose of this benchmark is for students to think about the magnitude of multi-digit numbers with decimals to round them to the nearest hundredth, tenth or whole number. In grade 5, the expectations for rounding are to the nearest hundredth and to digits other than the leading digit, e.g., round 29.834 to the nearest hundredth. Students have experience rounding whole numbers to the nearest 10, 100, or 1,000 in the Grade 3 Accelerated course (MA.4.NSO.1.4). Rounding skills continue to be important in later grades as students solve real-world problems with fractions and decimals (MA.6.NSO.2.3) and work with scientific notation (MA.8.NSO.1.4).

* Instruction develops some efficient rules for rounding fluently by building from the basic strategy of – “Is 29.834 closer to 20 or 30?” Number lines are effective tools for this type of thinking and help students relate the placement of numbers to benchmarks for rounding *(MTR.3.1, MTR.5.1)*.
* The expectation is that students have a deep understanding of place value and number sense in order to develop and use an algorithm or procedure for rounding. Additionally, students should explain and reason about their answers when they round and have numerous experiences using a number line and a place value chart as tools to support their work with rounding*.*

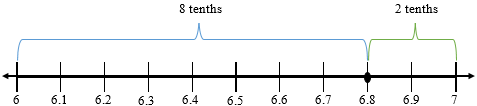
Common Misconceptions or Errors

* Students may confuse the place value to which they are rounding a number.

For example, when rounding 29.834 to the nearest tenth, they may have difficulty determining that 29.834 is between 29.8 and 29.9. The reliance on mnemonics, songs or rhymes during instruction can often confuse students further because they may lack the conceptual understanding for rounding decimals.

Strategies to Support Tiered Instruction

|  |
| --- |
| * Instruction includes using number lines and place value understanding to round multi-digit numbers with decimals to the nearest tenth or whole number.   + For example, students round 16.32 to the nearest tenth using a number line and place value understanding. The teacher explains that the endpoints of the number line will be represented using tenths, because we are rounding to the nearest tenth.  The teacher explains that there are three tenths in the number 16.32 and one more tenth would be four tenths. The teacher represents these endpoints on the number line as sixteen and three-tenths (16.3) and sixteen and four-tenths (16.4) while reminding students that 16.3 is equivalent to 16.30 and 16.4 is equivalent to 16.40. Additionally, the teacher explains that the mid-point on the number line can be labeled as sixteen and three-tenths, five-hundredths or sixteen and 35 hundredths (16.35). This midpoint is halfway between 16.3 and 16.4. The teacher asks students to plot 16.32 on the number line and discuss if it is closer to 16.3 or 16.4, explaining that 16.32 rounds to 16.3 because it is less than the midpoint of 16.35 and closer to 16.3 on the number line.   Number line |
| * + For example, students round 6.8 to the nearest whole number using a number line and place value understanding. The teacher explains that the endpoints of our number line will be represented using ones, because we are rounding to the nearest whole number. Also, the teacher explains that there are six ones in the number 6.8 and one more one would be seven ones. The teacher represents these endpoints on the number line as six ones (6) and seven ones (7). The midpoint on the number line is labeled as 6 ones and 5 tenths (6.5). This midpoint is halfway between 6 and 7. The teacher asks students to plot 6.8 on the number line and discuss if it is closer to six or seven, explaining that 6.8 rounds to seven because it is eight-tenths away from six and only two-tenths away from seven. It is also less than the midpoint of 6.5. |



Instructional Tasks

*Instructional Task 1 (MTR.3.1, MTR.6.1)*

Round 29.834 to the nearest whole number. Identify between which two whole numbers 29.834 lies on a number line.

*Instructional Task 2 (MTR.3.1, MTR.6.1)*

Round 29.834 to the nearest tenth. Identify between which two tenths 29.834 lies on a umber line.

*Instructional Task 3 (MTR.3.1, MTR.6.1)*

Round 29.834 to the nearest hundredth. Identify between which two hundredths 29.834 lies on a number line.

*Instructional Item 4*

For the number shown below, select all of the values for the missing digit that would cause 47.63 to round to 47.64 when round to the nearest hundredth.

47.63

1. 2
2. 4
3. 5
4. 6
5. 8

*Instructional Item 5*

Which of the following numbers rounds to 84.5 when rounded to the nearest tenth?

1. 84.42
2. 84.437
3. 84.526
4. 84.55

*Enrichment Task 1 (MTR.4.1)*

Explain how you can round 23.918 to the greatest place.

Enrichment Task 2 *(MTR.7.1)*

Rounded to the nearest dime, what is the greatest amount of money that rounds to $105.40? What is the least amount of money that rounds to $105.40? What strategy did you use to figure this out?

Instructional Items

*Instructional Item 1*

Which of the following are true about the number 104.029?

1. 104.029 rounded to the nearest whole number is 4.
2. 104.029 rounded to the nearest whole number is 104.
3. 104.029 rounded to the nearest tenth is 104.2.
4. 104.029 rounded to the nearest hundredth is 104.02.
5. 104.029 rounded to the nearest hundredth is 104.03.

*Enrichment Item 1 (MTR.3.1)*

Select all the numbers that round to 15.5 when rounded to the nearest tenth.

1. 15.04
2. 15.55
3. 15.508
4. 15.445
5. 15.0
6. 15.49

*Enrichment Item 2 (MTR.7.1)*

Students in Mrs. Gonzalez’s class measure the length of four ants. The students round the lengths to the nearest tenth. Which ant has a length that rounds to 0.8 inch?

1. 0.841 inch
2. 0.45 inch
3. 0.55 inch
4. 0.738 inch

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

**MA.4.NSO.2** *Build an understanding of operations with multi-digit numbers including decimals.*

### MA.4.NSO.2.3 & MA.5.NSO.2.1

Benchmark (Grade 4)

MA.4.NSO.2.3 Multiply two whole numbers, each up to two digits, including using a standard algorithm with procedural fluency.

Benchmark (Grade 5)

MA.5.NSO.2.1 Multiply multi-digit whole numbers including using a standard algorithm with procedural fluency.

Connecting Benchmarks/Horizontal Alignment

* MA.5.FR.2.2
* MA.5.AR.1.1
* MA.5.M.1.1
* MA.5.GR.3.1, MA.5.GR.3.2, MA.5.GR.3.3

Terms from the K-12 Glossary

* Equation
* Expression
* Factor

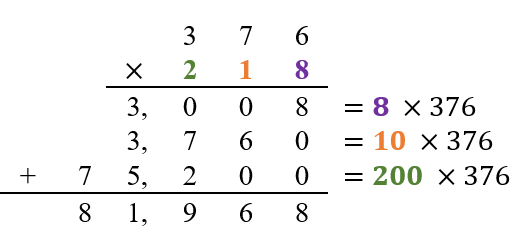
Vertical Alignment

|  |  |
| --- | --- |
| **Previous Benchmarks**   * MA. 3.NSO.2.2, MA.3.NSO.2.3, MA.3.NSO.2.4 | **Next Benchmarks**   * + MA.6.NSO.2.1 |

Purpose and Instructional Strategies

The purpose of this benchmark is for students in Grade 4 Accelerated is to demonstrate procedural fluency while multiplying multi-digit whole numbers. To demonstrate procedural fluency, students may choose the standard algorithm that works best for them and demonstrates their procedural fluency. A standard algorithm is a method that is efficient, generalizable (it correctly works no matter how many digits are involved) and accurate *(MTR.3.1)*. A standard algorithm is also generalizable meaning that it works no matter how many digits are involved. In the Accelerated Grade 3 course, students had experience multiplying two-digit by three-digit numbers using a method of their choice with procedural reliability (MA.4.NSO.2.2). In Grade 6, students will multiply and divide multi-digit numbers including decimals with fluency (MA.6.NSO.2.1).

* It is important to challenge students to explain the steps they follow when using a standard algorithm (i.e. regrouping, proper recording and placement of digits by place value).
* There is no limit on the number of digits for multiplication in Grade 5.
* When students use a standard algorithm, they should be able to justify why it works conceptually. Teachers can expect students to demonstrate how their algorithm works, for example, by comparing it to another method for multiplication *(MTR.6.1)*.
* Instruction includes helping students understand what each partial product in a standard algorithm represents.
  + For example, when using a standard algorithm to find the product of 376 and 218 students can use their understanding of place value and multiplication to write an equation for each partial product.



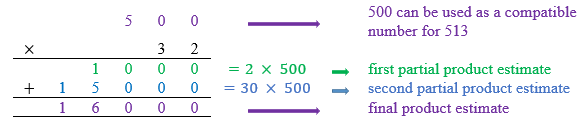
* Along with using a standard algorithm, students should estimate reasonable solutions before solving. Estimation helps students anticipate possible answers and evaluate whether their solutions make sense after solving.
* This benchmark supports students as they solve multi-step real-world problems involving combinations of operations with whole numbers (MA.5.AR.1.1).

Common Misconceptions or Errors

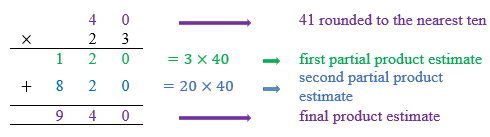
* Students that are taught a standard algorithm without any conceptual understanding will often make mistakes. For students to understand a standard algorithm or any other method, they need to be able to explain the process of the method they chose and why it works. This explanation may include pictures, properties of multiplication, decomposition, etc.
* Some students may struggle with this benchmark if they do not have a strong command of basic addition and multiplication facts.

Strategies to Support Tiered Instruction

* Instruction includes estimating reasonable values for partial products as well as final products.
  + For example, students make reasonable estimates for the partial products and final product for . Before using an algorithm, students can make estimates for partial products and final product to make sure that they are using the algorithm correctly and the answer is reasonable. First, students will estimate the first partial product by rounding 513 to the nearest hundred, 500, and multiplying by 2. When using an algorithm to solve the first partial product, the answer should be approximately 1,000. Next, students can estimate the second partial product by rounding 513 to 500 and multiplying by 30. When using an algorithm to solve the second partial product, it should be approximately 15,000. Finally, students can add the estimates for the partial products to find an estimate for the final product.

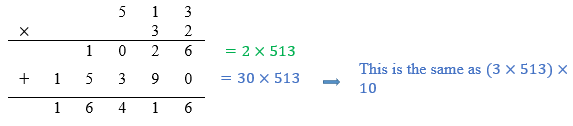
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* + For example, students make reasonable estimates for the partial products and final product for . Before using an algorithm, students can make estimates for our partial products and final product to make sure that they are using the algorithm correctly and the answer is reasonable. First, students will estimate the first partial product by rounding 41 to 40 and multiplying by 3. When using an algorithm to determine the first partial product, it should be approximately 120. Next, students will estimate the second partial product by rounding 41 to 40 and multiplying by 20. When using an algorithm to determine the second partial product, it should be approximately 800. Finally, students can add the estimates for the partial products to find an estimate for the final product.

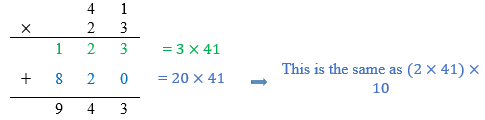
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* Instruction includes explaining and justifying mathematical reasoning while using a multiplication algorithm. Instruction includes determining if an algorithm was used correctly by analyzing any errors made and reviewing the reasonableness of solutions.

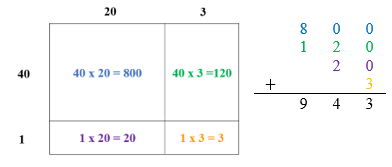
For example, students use an algorithm to determine and explain their thinking using place value understanding. Begin by multiplying 2 ones times 3 ones; students should recognize this equals 6 ones. Students can write the 6 ones under the line, in the ones place. Next, multiply 2 ones times 1 ten, which students should recognize this equals 2 tens. They can write the 2 tens under the line in the tens place. Then, multiply 2 ones times 5 hundreds, which equals 10 hundreds. Write the 10 hundreds under the line in the thousands and hundreds place because 10 hundred is the same as 1 thousand. Students should see that this gives the first partial product of 1,026. Now multiply the 3 ones by the 3 tens from 32; this equals 9 tens or 90. Record 90 below the first partial product of 1,026. Next, multiply the 1 ten by 3 tens, which equal 3 hundreds, and write the 3 in the hundreds place of the second partial product. Then, multiply the 5 hundreds times 3 tens, which equals 15 thousands. Students can write the 15 in the ten thousands and thousands place of our second partial product, noticing that the second partial product is 15,390. Finally, add the partial products to find the product of 16,416.



* + For example, have students use an algorithm to determine and explain their thinking using place value understanding. Explicit instruction could include “Begin by multiplying 3 ones times 1 one. This equals 3 ones. We will write the 3 ones under the line, in the ones place. Next, we will multiply 3 ones times 4 tens. This equals 12 tens. We will write the 12 tens under the line in the hundreds and tens place because 12 tens is the same as 1 hundred 2 tens. This gives us our first partial product of 123. Now we will multiply the 1 one by the 2 tens from 23. This equals 2 tens or 20. We will record 20 below our first partial product of 123. Next, we will multiply 2 tens times 4 tens, which equal 8 hundreds. We will write the 8 in the hundreds place of our second partial product. Our second partial product is 820. Finally, we add our partial products to get 943.”

****

* + For example, students solve using an area model and place value understanding and explain how each partial product is calculated and what it represents as they multiply using the area model. Then, students explain how the final product is calculated using the partial products from the area model.



Instructional Tasks

*Instructional Task 1 (MTR.4.1, MTR.5.1)*

Using the digits 1, 2, 3 and 4, arrange them to create two 2-digit numbers that when multiplied, will yield the greatest product.

*Instructional Task 2 (MTR.7.1)*

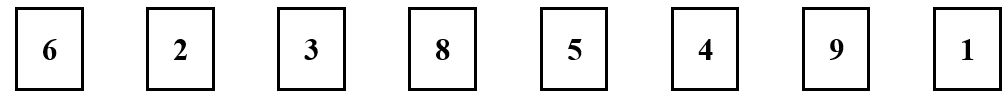
Maggie has three dogs. She buys a box containing 175 bags of dog food. Each bag weighs 64 ounces.

Part A. What is the total weight of the bags of dog food in ounces?

Part B. Maggie has a storage cart to transport the box that holds up to 750 pounds. Will the storage cart be able to hold the box? Explain.

*Instructional Task 3*

Select six cards from the set of cards below.



Part A. Use your cards to create a four-digit number and a two-digit number. Find the product of your four-digit and two-digit numbers.

Part B. Use your cards to create a three-digit number and another three-digit number. Find the product of your three-digit numbers.

Part C. How are your two products similar? How are they different?

*Instructional Task 4 (MTR.4.1, MTR.5.1)*

Using the digits 2, 3, 6, and 8 arrange them to create two 2-digit numbers that when multiplied, will yield the smallest product.

Instructional Items

*Instructional Item 1*

Select the expressions that have a product of 480.

*Instructional Item 2*

What is the product of 427 ?

*Instructional Item 3*

Select the expression that has a product of 255.

1. 15 X 17
2. 25 X 13
3. 25 X 25
4. 75 X 15

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

**MA.4.NSO.2** *Build an understanding of operations with multi-digit numbers including decimals.*

### MA.4.NSO.2.4 & MA.5.NSO.2.2

Benchmark (Grade 4)

MA.4.NSO.2.4 Divide a whole number up to four digits by a one-digit whole number with procedural reliability. Represent remainders as fractional parts of the divisor.

Benchmark Clarifications:

*Clarification 1:* Instruction focuses on helping a student choose a method they can use reliably.

*Clarification 2:* Instruction includes the use of models based on place value, properties of operations or the relationship between multiplication and division.

Benchmark (Grade 5)

MA.5.NSO.2.2 Divide multi-digit whole numbers, up to five digits by two digits, including using a standard algorithm with procedural fluency. Represent remainders as fractions.

*Example:* The quotient gives with remainder which can be expressed as .

Benchmark Clarifications:

*Clarification 1:* Within this benchmark, the expectation is not to use simplest form for fractions.

Connecting Benchmarks/Horizontal Alignment

* MA.5.FR.2.4
* MA.5.AR.1.1, MA.5.AR.1.3
* MA.5.M.1.1
* MA.5.GR.3.3

Terms from the K-12 Glossary

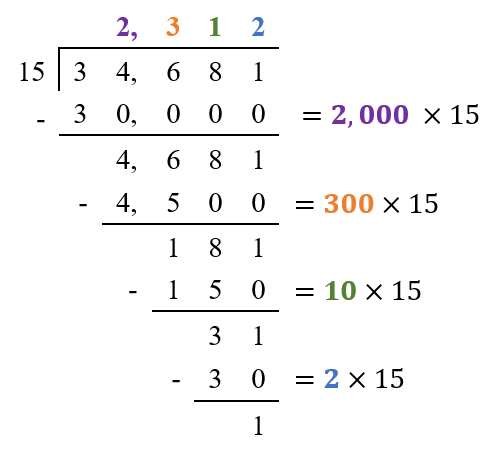
* Dividend
* Divisor
* Equation
* Expression
* Factor
* Quotient
* Whole Number

Vertical Alignment

|  |  |
| --- | --- |
| **Previous Benchmarks**   * MA.3.NSO.2.4 | **Next Benchmarks**   * + MA.6.NSO.2.1 |

Purpose and Instructional Strategies

The purpose of this benchmark is for students to demonstrate procedural fluency while dividing multi-digit whole numbers with up to 5-digit dividends and 2-digit divisors. To demonstrate procedural fluency, students in Grade 4 Accelerated may choose a standard algorithm that works best for them and demonstrates their procedural fluency. A standard algorithm is a method that is efficient, generalizable (it works correctly no matter how many digits are involved) and accurate *(MTR.3.1)* no matter how many digits are involved. In Grade 6, students will multiply and divide multi-digit numbers including decimals with fluency (MA.6.NSO.2.1).

* When students use a standard algorithm, they should be able to justify why it works conceptually. Teachers can expect students to demonstrate how their algorithm works, for example, by comparing it to another method for division *(MTR.6.1)*.
* When used efficiently, partial quotients are a suitable procedure for dividing multi-digit numbers. It can often be more reliable and efficient for students than the long division algorithm. Students can demonstrate fluency by skillfully using partial quotients and should be able to understand the full long division algorithm.
  + For example, students can use place value understanding, estimation and partial quotients to solve Students can begin by determining that the product gets them close to their dividend of 34,681 and continue by subtracting groups of 15 based place value and multiples of 15.
* In this benchmark, students are to represent remainders as fractions. In the benchmark example, the quotient of is represented as . Students should gain understanding that this quotient means that there are 3 full groups of 7 in 27, and the remainder of 6 represents of another group. Students are not expected to have mastery of converting between forms (fraction, decimal, percentage) until grade 6 but students should start to gain familiarity that fractions and decimals are numbers and can be equivalent (i.e., a remainder of is the same as 0.5). Writing remainders as fractions or decimals is acceptable. Similarly, students should be able to understand that a remainder of zero means that whole groups have been filled without any of the dividend remaining *(MTR.5.1, MTR.7.1)*.
* Along with using a standard algorithm, students should estimate reasonable solutions before solving. Estimation helps students anticipate possible answers and evaluate whether their solutions make sense after solving.
* When working with standard algorithms, it is important to note that when used well, partial quotients is a suitable procedure, even though it is technically not an “algorithm.” Partial quotients is not considered an algorithm because the steps are not fully prescribed; they involve some choice by the student. This procedure is often more reliable and efficient for a student than the long division algorithm that is considered “standard” in the U.S. Students can demonstrate fluency by skillfully using partial quotients, and any such student should be able to understand the full long division algorithm.
* This benchmark supports students as they solve multi-step real-world problems involving combinations of operations with whole numbers (MA.5.AR.1.1). In a real-world problem, students should interpret remainders depending on its context.

Common Misconceptions or Errors

* Students can make computational errors while using standard algorithms when they cannot reason why their algorithms work. In addition, they can struggle to determine where or why that computational mistake occurred because they did not estimate reasonable values for intermediate outcomes as well as for the final outcome. During instruction, teachers should expect students to justify their work while using their chosen algorithms and engage in error analysis activities to connect their understanding to the algorithm.

Strategies to Support Tiered Instruction

* Instruction includes estimating reasonable values for quotients when dividing by two-digit divisors.
  + For example, students make reasonable estimates for the quotient of . Before using an algorithm, students can estimate the quotient to make sure that they are using the algorithm correctly and the answer is reasonable. Students can use multiples of 24 and their understanding of multiplication and division to estimate the quotient. Students may want their estimate to be as close to 496 as possible. So, knowing that , they can state that . A reasonable estimate for the quotient would be 20 because 480 is close to 496.”
  + For example, students make reasonable estimates for the quotient of . Explicit instruction could include stating, “Before using an algorithm, we will estimate the quotient to make sure that we are using the algorithm correctly and our answer is reasonable. The divisor of 13 is close to 10 and the dividend of 94 is close to 90. So, we can use to estimate that our quotient should be close to 9.”
* Instruction includes explaining and justifying mathematical reasoning while using a division algorithm to divide by two-digit divisors. Instruction also includes determining if an algorithm was used correctly by analyzing any errors made and reviewing the reasonableness of solutions.
  + For example, the teacher connects place value with the partial quotients model to determine . Students should not just view the digits as individual numbers but connect individual digits with the value of that number (e.g., 496 is ). Instruction includes stating, “In this problem we are finding how many groups of 24 are in 496. We will subtract groups of 24 until we cannot subtract any more groups. The total number of groups that we can subtract is the quotient. We can subtract 10 groups of 24 two times, so the quotient is 20. We have a remainder of 16. The quotient is represented as 20 because we have 20 full groups of 24 in 496 and the remainder of 16 represents of another group.”

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  |  | 2 | 0 |  |  |
| 24 |  | 4 | 9 | 6 |  |  |
|  | - | 2 | 4 | 0 |  | 10 groups of 24 |
|  |  | 2 | 5 | 6 |  |  |
|  | - | 2 | 4 | 0 |  | 10 groups of 24 |
|  |  |  | 1 | 6 |  |  |

* + For example, connect place value with the partial quotients model to determine . Students should not just view the digits as individual numbers but connect individual digits with the value of that number (e.g., 94 is ). Instruction includes stating, “In this problem we are finding how many groups of 13 are in 94. We will subtract groups of 13 until we cannot subtract any more groups. The total number of groups that we can subtract is the quotient. We can subtract 7 groups of 13, so the quotient is 7. We have a remainder of 3. The quotient is represented as 7 because we have 7 full groups of 13 in 94 and the remainder of 3 represents of another group.”

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  | 7 |  |  |
| 13 |  | 9 | 4 |  |  |
|  | - | 2 | 6 |  | 2 groups of 13 |
|  |  | 6 | 8 |  |  |
|  | - | 2 | 6 |  | 2 groups of 13 |
|  |  | 4 | 2 |  |  |
|  | - | 2 | 6 |  | 2 groups of 13 |
|  |  | 1 | 6 |  |  |
|  | - | 1 | 3 |  | 1 group of 13 |
|  |  |  | 3 |  |  |

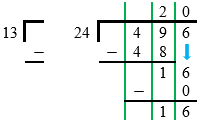
* + For example, students use an algorithm to solve and explain their thinking using place value understanding. Instruction includes stating, “In this problem we are finding how many groups of 24 are in 496. We will begin by dividing our largest place value first. Recognizing that the 4 represents 400, if your divide 400 by 24 the result will be less than 100, so the quotient won’t have any whole hundreds. Remember that 496 is the same as , so we will see how many groups of 24 are in . We can also think of this as . There are 20 groups of 24 in , that’s 2 times 10 groups, so we can place a 2 in the tens place of the quotient. Next, we will subtract (20 groups of 24 equal ) to find a difference of . We can combine this with the remaining in 496. We now have remaining from our original dividend of 496, this is not enough to make a group of 24. We have a remainder of 16. The quotient is represented as because we have 20 full groups of 24 in 496 and the remainder of 16 represents of another group. Our quotient of is close to our estimate of 20, this helps us determine that our answer is reasonable.”

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  | 2 | 0 |
| 24 |  | 4 | 9 | 6 |
|  |  | 4 | 8 |  |
|  |  |  | 1 | 6 |
|  |  |  |  | 0 |
|  |  |  | 1 | 6 |

* + For example, students use an algorithm to solve and explain their thinking using base ten blocks and place value understanding. Instruction includes stating, “In this problem we are finding how many groups of 13 are in 94. We will begin by dividing our largest place value first. How many groups of 13 are in ? Recognizing that the 9 represents 90, if you divide 90 by 13 the result will be less than 10, so the quotient won’t have any whole tens. Remember that 94 is the same as and , so we will see how many groups of 13 are in . We can also think of this as. There are 7 groups of 13 in . Next, we will subtract to find our remainder of 3. Our quotient is represented as because we have 7 full groups of 13 in 94 and the remainder of 3 represents of another group. Our quotient of is close to our estimate of 9, this helps us determine that our answer is reasonable.”

|  |  |
| --- | --- |
|  | **Division algorithm described above.**Base ten blocks modeling example above. |

* Instruction includes the use of place value columns to support place value understanding when using a division algorithm.
* Example:



Instructional Tasks

*Instructional Task 1 (MTR.7.1)*

The Magnolia Outreach organization is donating 6,924 pounds of rice to families in need. They pour all the rice into 15-pound containers.

Part A. How many containers will they fill completely if they use all the rice?

Part B. How many containers will they need for all of the rice?

Part C. Will the Magnolia Outreach organization completely fill all of the containers needed for all of the rice? If not, will the partially filled container be more or less than half-full? Explain how you know.

*Instructional Task 2*

Natalie says that 5,978 is not divisible by 24 a whole number of times. Is Natalie correct? Why or why not?

Instructional Items

*Instructional Item 1*

What is the quotient of ?

*Instructional Item 2*

Part A. What is the quotient of 65?

Part B. Write the remainder of 65, as a fraction.

*Instructional Item 3*

What is the quotient of ?

1. 20
2. 21
3. 201
4. 201

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

### MA.4.NSO.2.6

Benchmark (Grade 4)

MA.4.NSO.2.6 Identify the number that is one-tenth more, one-tenth less, one-hundredth more and one-hundredth less than a given number.

*Example:* One-hundredth less than 1.10 is 1.09.

*Example:* One-tenth more than 2.31 is 2.41.

Connecting Benchmarks/Horizontal Alignment

* MA.4.NSO.1.5
* MA.4.FR1.1, MA.4.FR.1.2
* MA.4.M.1.2
* MA.4.M.2.2

Terms from the K-12 Glossary

* Equation
* Expression

Vertical Alignment

|  |  |
| --- | --- |
| **Previous Benchmarks**   * MA.2.NSO.2.2 | **Next Benchmarks**   * MA.5.NSO.2.3, MA.5.NSO.2.4 |

Purpose and Instructional Strategies

The purpose of this benchmark is for students to develop an understanding of place value with tenths and hundredths in addition and subtraction.

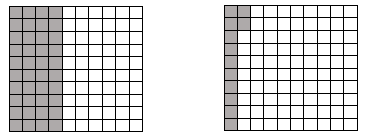
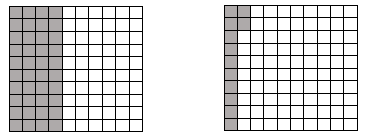
* This benchmark extends upon students’ thinking about 1 more/less from whole numbers to decimals. Students should continue using place value understanding to reason how adding and subtracting 1 tenth and 1 hundredth changes a number’s value.
* Teachers should use familiar manipulatives to help connect students’ exploration of decimals to whole numbers. These materials include base-ten blocks, tenths and hundredths charts (modeled after hundred charts students used in primary), and place value mats. During instruction, teachers model correct vocabulary consistently to describe decimals and expect the same from students (e.g., the number 1.09 is be read as “one *and* 9 hundredths”).
* In this initial exploration of decimal addition and subtraction, the expectation is to develop understanding using manipulatives, visual models, discussions, estimation and drawings, with the focus being on adding and subtracting 1 tenth and 1 hundredth. This prepares students for the broader exploration of adding and subtracting decimals in MA.4.NSO.2.7.

Common Misconceptions or Errors

* When using base-ten blocks, it is important to first identify the value of each block. Students may have preconceptions about relating units to ones, rods to tens, and flats to hundreds, which can be confusing when their values shift from whole numbers to decimals. Teachers should share the relationship between the blocks (each larger block is ten times larger the next smaller block) so that students understand they can be used flexibly.
* Students can struggle to understand that one-hundredth is smaller than one-tenth because of one hundred is larger than one ten. During instruction, emphasize that one-hundredth is smaller because it would require 100 hundredths to equal 1 whole and only 10 tenths to equal 1 whole.

Strategies to Support Tiered Instruction

* Instruction includes opportunities to model and represent decimals.
  + For example, if a 10 by 10 grid of 100 represents one whole, students shade in 0.4 on the grid using the appropriate language to connect “four-tenths” to the decimal 0.4. Then, students shade in what 0.12 represents. The teacher connects the language “twelve hundredths” to the decimal 0.12. Students compare the decimals using the visuals. This will help solidify the understanding that tenths are larger than hundredths. Using visuals will also connect the learning of one tenth more/less and one hundredth more/less.

**** ****

0.4 0.12

* Instruction includes building decimals with base ten blocks.
* For example, the teacher asks students to build 0.3 (three-tenths) and 0.4 (four-tenths).

|  |  |
| --- | --- |
| **Base ten blocks illustrating decimal above.** | **Base ten blocks illustrating decimal above.** |
| 0.3 | 0.4 |

Students physically see that 0.3 is one-tenth less than 0.4.

* + - During instruction, the teacher shares the relationship between the blocks (each larger block is ten times larger the next smaller block) to demonstrate that they can be used flexibly.
  + For example, emphasize that one-hundredth is smaller because it would require 100 hundredths to equal 1 whole and only 10 tenths to equal 1 whole.
* Opportunities for enrichment include extending student understanding of equivalent decimals. Plot decimals written in tenths, hundredths, and thousandths to build understanding of equivalence on a number line.

Instructional Tasks

*Instructional Task 1 (MTR.4.1)*

Vianna says that 1 tenth more than 3.9 is 4. Tracy says that 1 tenth more than 3.9 is 3.91. Who is correct? Explain how you know.

*Instructional Task 2 (MTR.4.1)*

Robert says that 5.21 is equivalent to 5.210. James says that they are not equivalent. Who is correct? Explain how you know.

*Enrichment Task 1 (MTR.1.1)*

Molly says, “0.30 is greater than 0.3 because 30 is greater than 3.” Do you agree? Explain.

*Enrichment Task 2 (MTR.7.1)*

Rachel runs the 100-meter race in 11.13 seconds. Courtney ran the same race in 11.12 seconds. Who had the faster time? How do you know?

Instructional Items

*Instructional Item 1*

What is one tenth more than 3.8?

What is one tenth less than 7?

What is one hundredth more than 15.29?

What is one hundredth less than 7?

*Instructional Item 2*

*What is the equivalent decimal to 4.2?*

*What is one tenth less than 3?*

*What is one hundredth more than 4.23?*

*What is one hundredth less than 2.80?*

*Enrichment Item 1 (MTR.6.1)*

Will the product of be greater than, less than, or the same as 73.8? Explain how you know.

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

**MA.5.NSO.2** *Add, subtract, multiply and divide multi-digit numbers.*

### MA.5.NSO.2.1

### See Benchmark MA.4.NSO.2.3

### MA.5.NSO.2.2

### See Benchmark MA.4.NSO.2.4

### MA.5.NSO.2.3

### See Benchmark MA.4.NSO.2.7

### MA.5.NSO.2.4

Benchmark

MA.5.NSO.2.4 Explore the multiplication and division of multi-digit numbers with decimals to the hundredths using estimation, rounding and place value.

*Example:* The quotient of 23 and 0.42 can be estimated as a little bigger than 46 because 0.42 is less than one-half and 23 times 2 is 46

Benchmark Clarifications:

*Clarification 1:* Estimating quotients builds the foundation for division using a standard algorithm.

*Clarification 2:* Instruction includes the use of models based on place value and the properties of operations.

Connecting Benchmarks/Horizontal Alignment

* MA.5.NSO.1.1/1.2/1.3/1.4/1.5
* MA.5.FR.2.3
* MA.5.AR.2.2/2.3
* MA.5.M.1.1
* MA.5.M.2.1
* MA.5.GR.2.1

Terms from the K-12 Glossary

* Equation
* Expression

Vertical Alignment

|  |  |
| --- | --- |
| **Previous Benchmarks**   * MA.4.NSO.2.7 | **Next Benchmarks**   * + MA.6.NSO.2.1 |

Purpose and Instructional Strategies

The purpose of this benchmark is for students to explore multiplication and division of multi-digit numbers with decimals using estimation, rounding, place value, and exploring the relationship between multiplication and division. This benchmark connects to the work students did earlier in the Grade 4 Accelerated course with addition and subtraction of decimals (MA.4.NSO.2.7). Students achieve procedural fluency with multiplying and dividing multi-digit numbers with decimals in grade 6 (MA.6.NSO.2.1)

* Instruction of this benchmark focuses on number sense to help students develop procedural reliability while multiplying and dividing multi-digit numbers with decimals.
* Students should have practice solving multiplication and division equations with decimals using strategies based on place value and the properties of operations.
* During instruction, students should explore how the products and quotients of whole numbers relate to decimals.
  + For example, if students know the product of and the quotient of , then they can reason through or through place value relationships. Classroom discussions should allow students to explore these patterns and use them to estimate products and quotients *(MTR.4.1, MTR.6.1)*.
* Instruction includes dividing decimals as if all of the numbers were whole numbers and then using estimation to place the decimal point.
  + For example, to solve a reasonable estimate of 12 could be found by determining how many groups of 3 *tenths* can be formed from the 36 *tenths* in 3.69. Dividing 3.69 by 0.3 as if they are whole numbers would result in a quotient of 123. The estimate of 12 would be used to place the decimal after the 2 in 123 resulting in a quotient of 12.3 for
* Instruction makes the connection between multiplication of fractions and decimals to help students reason about the magnitude of their products and determine correct placement of the decimal point.

For example, can be written as to make connections to multiplying fractions and help students understand why their product results in a decimal to the thousandths. For this problem, so = 0.052.

* Teachers should connect what students know about place value and fractions.
* For example, because students know that multiplying a number by one-fourth will result in a product that is less, multiplying a number by 0.25 (its decimal equivalence) will also result in a less product. In division, dividing a number by one-fourth and 0.25 will result in a greater quotient. Continued work in this benchmark will help students to generalize patterns in multiplication and division of whole numbers and fractions *(MTR.5.1)*.
* Models that help students explore the multiplication and division of multi-digit numbers with decimals include base ten representations (e.g., blocks) and place value mats

Common Misconceptions or Errors

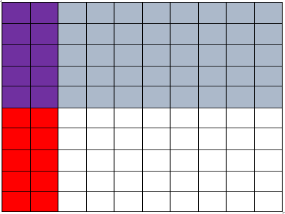
* Students may not understand the reasoning behind the placement of the decimal point in the product or quotient. Modeling and exploring the relationships between place value will help students gain understanding.
* Students can confuse that multiplication always results in a larger product, and that division always results in a smaller quotient. Through classroom discussion, estimation and modeling, classroom work should address this misconception.

Strategies to Support Tiered Instruction

* Instruction includes opportunities to predict and explain the relative size of the product of two decimals. Students use models to check their prediction and solve. The teacher guides students to connect that multiplying a given number by a number less than one will result in a smaller number, and that multiplying a given number by a number greater than one will result in a larger number.
  + For example, students solve the following problem . Students should reason about the size of the decimals and connect it back to their fraction understanding and think about the multiplication sign signaling “groups of.” This expression could be interpreted as 0.2 “of” 0.5. This will help with the misconception of multiplying equals a larger product.

The picture below illustrates the product of 0.2 and 0.5. If the entire square is 1 unit, the gray region represents 0.5 units, and the red region represents 0.2 units. The overlap in purple contains 10 small squares, each of which represents 0.01 units. Therefore, the overlap portion contains units. The overlap portions show a 0.2 by 0.5 rectangle, so the number of units it contains is the product 0.2 and 0.5.

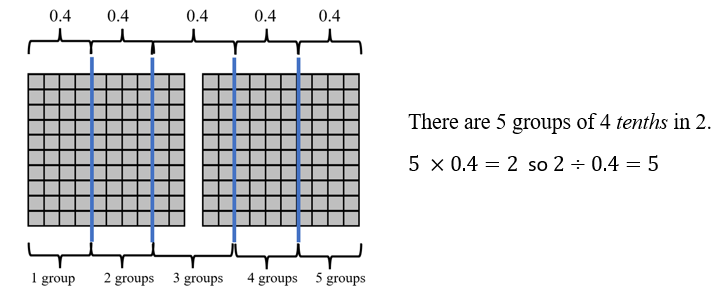
**0.5**



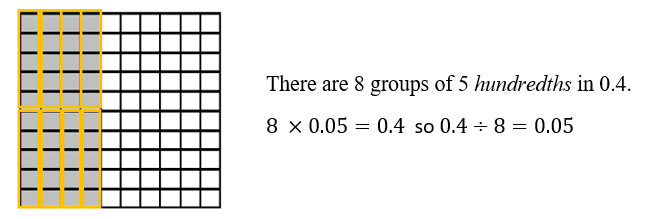
**0.2**

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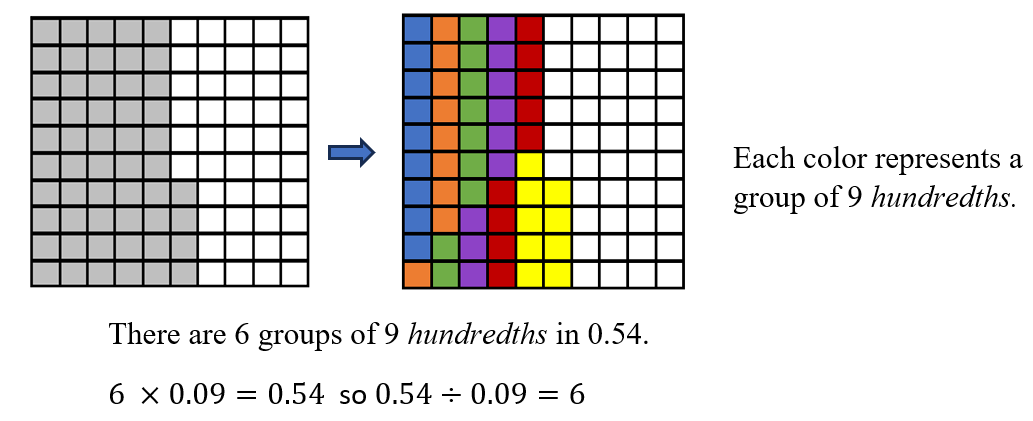
* Instruction includes opportunities to predict and explain the relative size of the quotient of a decimal and a whole number or two decimals. Students use models based on place value to check their prediction and solve. The teacher guides students to connect their quotients to the number of groups or size of the groups based on the magnitude of the divisor and dividend.
  + For example, students can use a decimal grid to solve by using the following model:

****

* + For example, students can use a decimal grid to solve 0.4 by using the following model:

****

* For example, students can use a decimal grid to solve 0.54 by using the following model:



Instructional Tasks

*Instructional Task 1 (MTR.4.1)*

What is the same about the products of these expressions? What is the difference? Explain.

*Instructional Task 2 (MTR.4.1)*

What is the same about the quotients of these expressions? What is the difference? Explain.

*Instructional Task 3 (MTR.5.1)*

How can you use to help you find the product of ? Explain.

Instructional Items

*Instructional Item 1*

Raul reasons that the product of will be greater than 41 and less than 82. Explain whether or not his conclusion is reasonable.

*Instructional Item 2*

What is the quotient of ?

*Instructional Item 3*

What is the product of ?

*Instructional Item 4*

What is the quotient of 3.68?

*Instructional Item 5*

Which of the following equations has a missing value of 4.8? Select all that apply.

1. \_\_\_\_\_
2. \_\_\_\_\_
3. \_\_\_\_\_
4. \_\_\_\_\_\_
5. \_\_\_\_\_\_

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

### MA.5.NSO.2.5

Benchmark

MA.5.NSO.2.5 Multiply and divide a multi-digit number with decimals to the tenths by one-tenth and one-hundredth with procedural reliability.

*Example:* The number divided by can be thought of as to determine the quotient is .

Benchmark Clarifications:

*Clarification 1:* Instruction focuses on the place value of the digit when multiplying or dividing.

Connecting Benchmarks/Horizontal Alignment

* MA.5.NSO.1.1/1.2/1.3/1.4
* MA.5.FR.2.3
* MA.5.AR.2.2/2.3
* MA.5.M.1.1
* MA.5.M.2.1
* MA.5.GR.2.1

Terms from the K-12 Glossary

* Equation
* Expression

Vertical Alignment

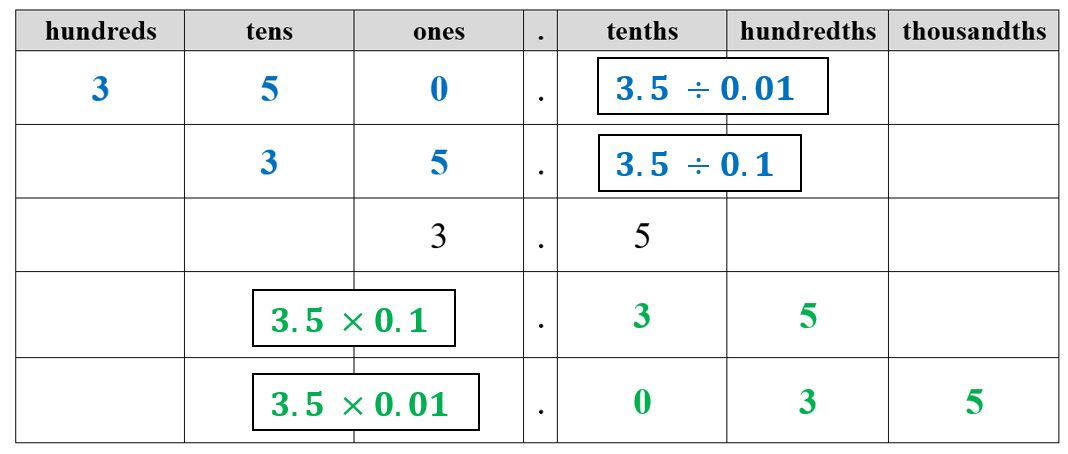
|  |  |
| --- | --- |
| **Previous Benchmarks**   * MA.4.NSO.2.6 | **Next Benchmarks**   * + MA.6.NSO.2.1 |

Purpose and Instructional Strategies

The purpose of this benchmark is for students to multiply multi-digit numbers with decimals to the tenths by 0.1 and by 0.01 with procedural reliability. Procedural reliability refers to the ability for students to develop an accurate, reliable method that aligns with a student’s understanding and learning style. Fluency of multiplying and dividing multi-digit whole numbers with decimals is not expected until Grade 6 (MA.6.NSO.2.1).

* When multiplying and dividing, students should continue to use the number sense strategies built in MA.5.NSO.2.4 (estimation, rounding and exploring place value relationships). Using these strategies will help students predict reasonable solutions and determine whether their solutions make sense after solving.
* During instruction, students should see the relationship between multiplying and dividing multi-digit numbers with decimals to multiplying and dividing by whole numbers. Students extend their understanding to generalize patterns that exist when multiplying or dividing by 10 or 100 *(MTR.5.1)*.
* Instruction includes the language that the “digits shift” relative to the position of the decimal point as long as there is an accompanying explanation. An instructional strategy that helps students see this is by putting digits on sticky notes or cards and showing how the values shift (or the decimal point moves) when multiplying by a power of ten.

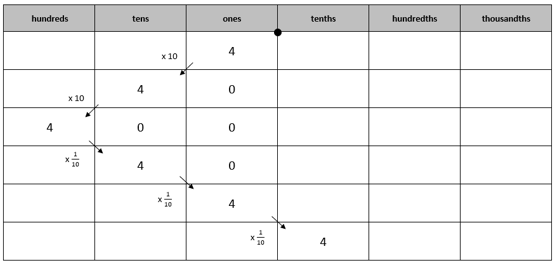
For example, a teacher could show one card with a 3 and another with a 5, and place them on the left and right of a decimal point on a blank place value chart. The teacher could then ask students to multiply by ten and shift both digits one place left to show the equation . They could ask students to multiply by and show that . Instruction emphasizes that the decimal point indicates the transition from the ones to the tenths place and that the digits shift when multiplying or dividing by 0.1 or 0.01 *(MTR.2.1, MTR.4.1,MTR.5.1)*.

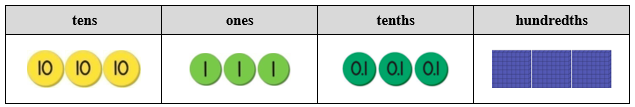


Common Misconceptions or Errors

* Students can confuse that multiplication always results in a larger product, and that division always results in a smaller quotient. Through classroom discussion, estimation and modeling, classroom work should address this misconception.

Strategies to Support Tiered Instruction

* Instruction includes the use of a place value chart to demonstrate how the value of a digit changes if the digit moves one place to the left or right. Instruction includes using place value understanding to make the connections between of, ten times less and dividing by 10. Also, the place value chart can be used to demonstrate that the decimal point marks the transition between the ones place and the tenths place.
  + For example, students multiply 4 by 10, then record 4 and the product of 40 in a place value chart. This process is repeated by multiplying 40 by 10. The teacher asks students to explain what happens to the digit 4 each time it is multiplied by 10. Next, the teacher explains that multiplying by is the same as dividing by 10. Students multiply 400 by and record the product in their place value chart. The process is repeated, multiplying 40 and 4 by . Students explain how the value of the 4 changed when being multiplied by 10 and .
  + ****
* Instruction includes opportunities to use models such as place value disks to demonstrate how the value of a digit changes if the digit moves one place to the left or right. A place value chart can be used with the models to support place value understanding and demonstrate that the decimal point marks the transition between the ones place and the tenths place. Instruction includes using place value understanding to make connections between of, ten times less and dividing by 10.
* For example, the teacher uses a familiar context such as money, asking students to explain the value of each digit in $33.33. Then, students represent 33.33 in a place value chart using place value disks. Students compare the value of the whole numbers, (3 dollars and 30 dollars), then move to comparing 0.3 and 0.03 (30 cents and 3 cents). The teacher asks, “How does the value of the three in the hundredths place compare to the value of the three in the tenths place?” and explains that the three in the hundredths place is the value of the three in the tenths place and that multiplying by is the same as dividing by 10.



Instructional Tasks

*Instructional Task 1 (MTR.7.1)*

Part A. What is times 15?

Part B. How many dimes are in $1.50?

Part C. Write a division equation to represent how many dimes are in $1.50.

Part D. Write a related multiplication equation to represent how many dimes are in $1.50.

*Instructional Task 2 (MTR.7.1)*

Part A. What is times 60?

Part B. How many pennies are in $0.60?

Part C. Write a division equation to represent how many pennies are in $0.60.

Part D. Write a related multiplication equation to represent how many pennies are in $0.60.

Instructional Items

*Instructional Item 1*

Which compares the products of and correctly?

* 1. The product of is 100 times less than the product of .
  2. The product of is 10 times less than the product of .
  3. The product of is 100 times more than the product of .
  4. The product of is 10 times more than the product of .

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

## Fractions

**MA.4.FR.1** *Develop an understanding of the relationship between different fractions and the relationship between fractions and decimals.*

### MA.4.FR.1.2

Benchmark

MA.4.FR.1.2 Use decimal notation to represent fractions with denominators of 10 or 100, including mixed numbers and fractions greater than 1, and use fractional notation with denominators of 10 or 100 to represent decimals.

Benchmark Clarifications:

*Clarification 1:* Instruction emphasizes conceptual understanding through the use of manipulatives visual models, number lines or equations.

*Clarification 2:* Instruction includes the understanding that a decimal and fraction that are equivalent represent the same point on the number line and that fractions with denominators of 10 or powers of 10 may be called decimal fractions.

Connecting Benchmarks/Horizontal Alignment

* MA.4.NSO.1.5
* MA.4.NSO.2.6/2.7
* MA.4.M.1.1/1.2
* MA.4.M.2.2

Terms from the K-12 Glossary

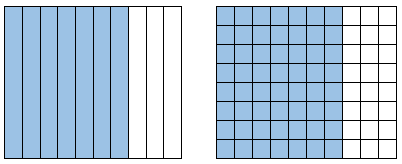
Vertical Alignment

|  |  |
| --- | --- |
| **Previous Benchmarks**   * MA.3.FR.2.2 | **Next Benchmarks**   * MA.6.NSO.3.5 |

Purpose and Instructional Strategies

The purpose of this benchmark is to connect fractions to decimals. Students extend their understanding of fraction equivalence (MA.3.FR.2.2) to include decimal fractions with denominators of 10 or 100. The connection will be continued in grade 6 (MA.6.NSO.3.5) and completed in grade 7 (MA.7.NSO.1.2).

* Instruction should help students understand that decimals are another way to write fractions. The place value system developed for whole numbers extends to fractional parts represented as decimals. The concept of one whole used in fractions is extended to models of decimals. It is important that students make connections between fractions and decimals in models.
* Instruction should provide visual fraction models of tenths and hundredths, number lines and equations so that students can express a fraction with a denominator of 10 as an equivalent fraction with a denominator of 100.
* Students reinforce understanding that the names for decimals match their fraction equivalents (e.g., , , , , , and are all equivalent).



* This benchmark is a connection point to the metric system and will be explored in MA.4.M.1.2.
* To help students understand decimal fractions, instruction can use models as seen above and name the value using the fraction representation to then extend the place value chart including the decimal point, tenths, and hundredths.
* For example, one whole divided into ten equal parts creates the tenths place and tenths divided into ten equal parts results in hundredths. This is an opportunity to make connections to MA.4.NSO.1.1.

Common Misconceptions or Errors

* Students often confuse decimals such as 6 tenths and 6 hundredths. Students should use models and explain their reasoning to develop their understanding about the connections between fractions and decimals.
* Some students may not understand that fractions and decimals are different presentations of the same thing. Number lines and other visual models will help students gain a better understanding of this concept.

Strategies to Support Tiered Instruction

* Instruction includes building fractions and their decimals equivalents using base ten blocks.
  + For example, students build “two-tenths” and “twenty hundredths” with base ten blocks while using vocabulary that will help students see the decimal connection as well. Students realize that the numbers have the same value.

|  |  |
| --- | --- |
| **Two tens rods.** | **Twenty units cubes.** |
| or 0.2 | or 0.20 |

* Opportunities for enrichment include representing numbers greater than 1 as fractions over ten and one hundred. Instruction includes creating area models or number lines to determine fraction and decimal equivalence.
* For example, the teacher can provide the fraction 73/10. Students build equivalent fractions to equal 73/10 such as 70/10 + 3/10/ Students understand that 70/10 is equivalent to 7 wholes, and 3/10 is equivalent to 3 tenths, therefore the decimal notation for 73/10 is 7.3.

Instructional Tasks

*Instructional Task 1 (MTR.6.1)*

Read the following numbers and use the benchmark fractions to place them on the number line.

1. 0.8
2. 0.32
3. 0.6
4. 0.17

Number line.

*Instructional Task 2*

Part A

Write each fraction using decimal notation.

\_\_\_\_\_\_\_\_ \_\_\_\_ \_\_\_\_

Part B

Which numbers in Part A are equivalent? Explain your thinking using words or models.

*Enrichment Task 1 (MTR.3.1)*

Represent the following fractions as decimals

a. 734/100

b. 952/100

c. 56/10

*Enrichment Task 2 (MTR.2.1)*

Draw an area model to represent the following decimal in tenths and hundredths

**0.57**

Instructional Items

*Instructional Item 1*

A value is shown.

What is the value in decimal form?

1. 0.25
2. 2.05
3. 2.5
4. 25.100

*Instructional Item 2*

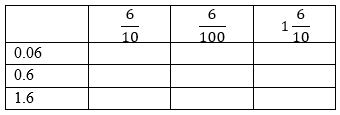
A value is shown.

*3.2*

What is the value in fraction form?

*Instructional Item 3*

Match each decimal to its representation as a fraction or mixed number.



\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

***MA.4.FR.2*** *Build a foundation of addition, subtraction and multiplication operations with fractions.*

### MA.4.FR.2.4 & MA.5.FR.2.2

Benchmark (Grade 4)

MA.4.FR.2.4 Extend previous understanding of multiplication to explore the multiplication of a fraction by a whole number or a whole number by a fraction.

*Example:* Shanice thinks about finding the product by imagining having 8 pizzas that she wants to split equally with three of her friends. She and each of her friends will get 2 pizzas since .

*Example:* Lacey thinks about finding the product by imagining having 8 pizza boxes each with one-quarter slice of a pizza left. If she put them all together, she would have a total of 2 whole pizzas since which is equivalent to 2

Benchmark Clarifications:

*Clarification 1:* Instruction includes the use of visual models or number lines and the connection to the commutative property of multiplication. Refer to Properties of Operation, Equality and Inequality (Appendix D).

*Clarification 2:* Within this benchmark, the expectation is not to simplify or use lowest terms.

*Clarification 3:* Fractions multiplied by a whole number are limited to less than 1. All denominators are limited to 2, 3, 4, 5, 6, 8, 10, 12, 16, 100.

Benchmark (Grade 5)

MA.5.FR.2.2 Extend previous understanding of multiplication to multiply a fraction by a fraction, including mixed numbers and fractions greater than 1, with procedural reliability.

Connecting Benchmarks/Horizontal Alignment

* MA.5.NSO.2.1/2.4
* MA.5.AR.1.2
* MA.5.FR.2.3
* MA.5.GR.2.1

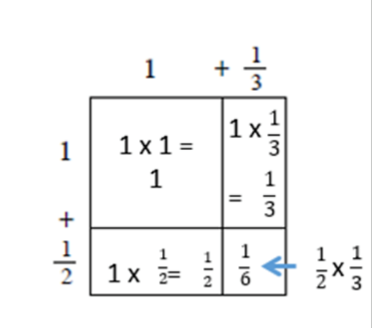
Terms from the K-12 Glossary

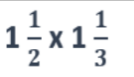
Vertical Alignment

|  |  |
| --- | --- |
| **Previous Benchmarks**   * MA.3.NSO.2.2 * MA.3.FR.1.2 | **Next Benchmarks**   * MA. 6.NSO.2.2 |

Purpose and Instructional Strategies

The purpose of this benchmark is for students in Grade 4 Accelerated to learn strategies to multiply a whole number times a fraction and a fraction times a whole number as well as multiply two fractions. Procedural fluency will be achieved in grade 6 (MA.6.NSO.2.2).

* During instruction, students are expected to multiply fractions including proper fractions, improper fractions (fractions greater than 1), and mixed numbers efficiently and accurately.
* Visual fraction models (area models, tape diagrams, number lines) should be used and created by students during their work with this benchmark *(MTR.2.1)*. Visual fraction models should show how a fraction is partitioned into parts that are the same as the product of the denominators.



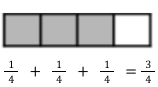
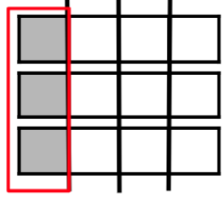
* When exploring an algorithm to multiply fractions (), make connections to an accompanying area model. This will help students understand the algorithm conceptually and use it more accurately.
* Instruction includes students using equivalent fractions to simplify answers; however, putting answers in simplest form is not a priority.

Common Misconceptions or Errors

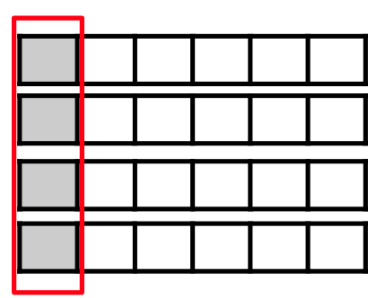
* Students may believe that multiplication always results in a larger number. Using models when multiplying with fractions will enable students to generalize about multiplication algorithms that are based on conceptual understanding (*MTR.5.1*).
* Students can have difficulty with word problems when determining which operation to use, and the stress of working with fractions makes this happen more often.
* For example, “Mark has yards of rope and he gives a third of the rope to a friend. How much rope does Mark have left?” expects students to first find of or multiply x , and then to find the difference to find how much Mark has left. On the other hand, “Mark has yards of rope and gives yard of rope to a friend. How much rope does Mark have left?” only requires finding the difference -

Strategies to Support Tiered Instruction

* Instruction includes real-world situations aligned with the content. The teacher provides a multiplication expression with real-world context and items to represent the situation to make connections.
  + For example, the teacher provides the student with the following situation: “The theater class has to make costumes that require foot of ribbon. They need to make 8 costumes. How many feet of ribbon will they need total?” The teacher provides ribbon already cut into foot pieces and has students model the problem. Students then write out the repeated addition problem to match or 2 feet.
  + For example, the teacher provides the student with the following situation: “The theater class has 8 feet of ribbon. They need to make 4 costumes and they want to use up all of the ribbon. How many feet of ribbon does each costume use?”The teacher will provide the student with a piece of ribbon 8 feet long and have them mark it off into 4 equal sections and find the length of 1 section (2 feet).
* Teacher provides models that represent multiplication expressions to represent the Commutative Property of Multiplication and has students explain the difference in meaning.
  + For example, the teacher provides the student with the following models.

* The teacher provides models and asks students which expression it models.
  + For example, the teacher provides students with the following model and asks if it represents or .



* Instruction involves real-world examples and models which allow students to see that multiplication does not always result in a larger number.
  + For example, the teacher provides the problem: “Tau has of the lasagna pan leftover from the party in the refrigerator. He eats one half of the leftovers for dinner. How much of the lasagna did he eat for dinner?” This can be written as or “of”

|  |  |
| --- | --- |
| Fraction bar model. | of the lasagna pan leftover in the refrigerator. |
| Fraction bar model. | of the lasagna leftover in the refrigerator. |

When students think about what is left in the refrigerator now, they must think in terms of the whole pan of lasagna. Tau didn’t eat half the pan; he ate half of the portion that was left. So how much of the whole pan did he eat?

|  |  |
| --- | --- |
| area model | of the lasagna pan. |

Instructional Tasks

*Instructional Task 1 (MTR.4.1, MTR.7.1)*

Part A. Maritza has cups of cream cheese. She uses of the cream cheese for a banana pudding recipe. After she uses it for the recipe, how much cream cheese will Maritza have left?

Part B. To find out how much cream cheese she used, Maritza multiplied as Will this method work? Why or why not.

Part C. What additional step is required to find how much cream cheese she has left?

*Instructional Task 2*

Draw a model to show

Then find the product. If possible, write the product as a mixed number

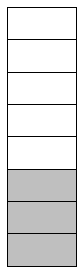
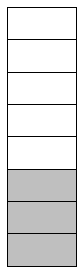
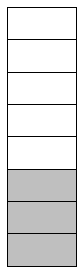
Instructional Items

*Instructional Item 1*

What is the product of ?

*Instructional Item 2*

*The model shows*



What is the product?

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

***MA.5.FR.1*** *Interpret a fraction as an answer to a division problem.*

### MA.5.FR.1.1

Benchmark

MA.5.FR.1.1 Given a mathematical or real-world problem, represent the division of two whole numbers as a fraction.

*Example:* At Shawn’s birthday party, a two-gallon container of lemonade is shared equally among 20 friends. Each friend will have of a gallon of lemonade which is equivalent to one-tenth of a gallon which is a little more than 12 ounces.

Benchmark Clarifications:

*Clarification 1:* Instruction includes making a connection between fractions and division by understanding that fractions can also represent division of a numerator by a denominator.

*Clarification 2:* Within this benchmark, the expectation is not to simplify or use lowest terms.

*Clarification 3:* Fractions can include fractions greater than one.

Connecting Benchmarks/Horizontal Alignment

* MA.4.NSO.2.4
* MA.5.NSO.2.2
* MA.5.AR.1.1
* MA.5.GR.3.3
* MA.5.DP.1.2

Terms from the K-12 Glossary

Vertical Alignment

|  |  |
| --- | --- |
| **Previous Benchmarks**   * MA.4.NSO.2.4 | **Next Benchmarks**   * MA.6.NSO.2.2 |

Purpose and Instructional Strategies

The purpose of this benchmark is for students to understand that a division expression can be written as a fraction by explaining their thinking when working with fractions in various contexts. This builds on the understanding developed earlier in the Grade 4 Accelerated course that remainders are fractions and prepares students for the division of fractions in grade 6 (MA.6.NSO.2.2).

* When students read as “,” they should be taught that can also be interpreted as “5 divided by 8,” where 5 represents the numerator and 8 represents the denominator of the fraction ( and refers to 5 wholes divided into 8 equal parts.
* Teachers can activate students’ prior knowledge of fractions as division by using fractions that represent whole numbers (e.g., ). Familiar division expressions help build students’ understanding of the relationship between fractions and division *(MTR.5.1)*.
* During instruction, provide examples accompanied by area and number line models.
* When solving mathematical or real-world problems involving division of whole numbers and interpreting the quotient in the context of the problem, students will be able to represent the division of two whole numbers as a mixed number, where the remainder is the fractional part’s numerator and the size of a group is its denominator (for example, equals 5 which is the number of size 3 groups you can make from 17 objects including the fractional group). Students should demonstrate their understanding by explaining or illustrating solutions using visual fraction models or equations

Common Misconceptions or Errors

* Students can believe that the fraction bar represents subtraction in lieu of understanding that the fraction bar represents division.
* Students can have the misconception that division always result in a smaller number.
* Students can presume that dividends must always be greater than divisors and, thus, reorder when representing a division expression as a fraction. Show students examples of fractions with greater numerators and greater denominators to create a division equation.

Strategies to Support Tiered Instruction

* Instruction includes making the connection to models and tools previously used to understand division as equal groups or sharing, but now as a fraction in a real-world context.
  + For example, “Eight friends share four brownies” can be represented as . This means that can be represented using the model below. Four is divided into 8 equal parts, each part is of the brownie.

**Model for division problem above.**

* + - Connecting the real-world application to the fraction will help students understand that the fraction really means division.
* Instruction includes making the connection to models and tools previously used to understand division as equal groups or sharing, but now as a fraction in a real-world context.
  + For example, “Marcos has 8 toy cars that he wants to put into 4 boxes equally. How many cars can go in each box?” can be shown using a model of 8 wholes divided into 4 groups. The quotient would be the total number of pieces in each group. The model below would show that . This can also be expressed as .

**Model for division problem above**

* Instruction includes examples of fractions with greater numerators and greater denominators to create a division equation

Instructional Tasks

*Instructional Task 1 (MTR.7.1)*

Create a real-world division problem that results in an answer equivalent to .

Instructional Items

*Instructional Item 1*

Write a mixed number that is equivalent to

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

**MA.5.FR.2** *Perform operations with fractions.*

### MA.5.FR.2.1

Benchmark

MA.5.FR.2.1 Add and subtract fractions with unlike denominators, including mixed numbers and fractions greater than 1, with procedural reliability.

*Example:* The sum of and can be determined as , or by using different common denominators or equivalent fractions.

Benchmark Clarifications:

*Clarification 1:* Instruction includes the use of estimation, manipulatives, drawings or the properties of operations.

*Clarification 2:* Instruction builds on the understanding from previous grades of factors up to 12 and their multiples.

Connecting Benchmarks/Horizontal Alignment

* MA.5.NSO.2.3
* MA.5.AR.1.2
* MA.5.GR.2.1

Terms from the K-12 Glossary

Vertical Alignment

|  |  |
| --- | --- |
| **Previous Benchmarks**   * MA.4.FR.1.3 * MA.4.FR.2.1/2.2 | **Next Benchmarks**   * MA. 6.NSO.2.3 |

Purpose and Instructional Strategies

The purpose of this benchmark is for students to understand that when adding or subtracting fractions with unlike denominators, equivalent fractions are generated to rewrite the fractions with like denominators, with which students have experience from the Grade 3 Accelerated course (MA.4.FR.2.2). Procedural fluency will be achieved in grade 6 (MA.6.NSO.2.3).

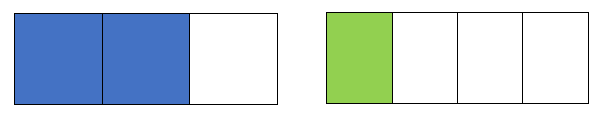
* During instruction, have students begin with expressions with two fractions that require the rewriting of one of the fractions (where one denominator is a multiple of the other, like or ) and progress to expressions where both fractions must be rewritten (where denominators are not multiples of one another, like or ). In doing so, students can explore how both fractions need like denominators to make addition and subtraction easier. Once students have stronger conceptual understanding, expressions requiring adding or subtracting 3 or more numbers should be included in instruction.
* It is important for students to practice problems that include various fraction models as students may find that a circular model might not be the best model when adding or subtracting fractions because of the difficulty in partitioning the pieces so they are equal *(MTR.2.1)*.
* When students use an algorithm to add or subtract fractions, encourage students’ use of flexible strategies.
  + For example, students can use a partial sums strategy when adding by adding the whole numbers together first before adding the fractional parts and regrouping when necessary.
* Mental computations and estimation strategies should be used to determine the reasonableness of solutions.
* For example, when adding , students could reason that the sum will be greater than 6 because the sum of the whole numbers is 5 and the sum of the fractional parts in the mixed numbers will be greater than 1. Keep in mind that estimation is about getting reasonable solutions and not about getting exact solutions, therefore allow for flexible estimation strategies and expect students to justify them.
* Although not required, instruction may include students using equivalent fractions to simplify answers.

Common Misconceptions or Errors

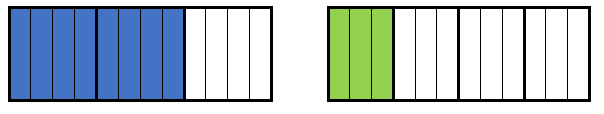
* Students can carry misconceptions from the Grade 3 Accelerated course about adding and subtracting fractions and understanding why the denominator remains the same. Emphasize the use of area and number line models, and present expressions in numeral-word form to help understand that the denominator is the unit.
  + For example, “ is equal to how many ?”
* Students often try to use different models when adding, subtracting or comparing fractions.
  + For example, they may use a circle for thirds and a rectangle for fourths, when comparing fractions with thirds and fourths.
* Remind students that the representations need to be from the same whole models with the same shape and same size. In a real-world problem, this often looks like same units.
  + For example, “Trey has cups of water and Rachel has cups of water. How many cups of water do they have?”

Strategies to Support Tiered Instruction

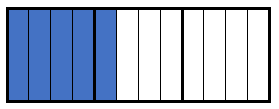
* Instruction includes concrete models and drawings that help solidify understanding that when adding and subtracting with unlike denominators, the value of the fractional parts remains the same.
  + For example, students create a model for each of the fractions in the problem .



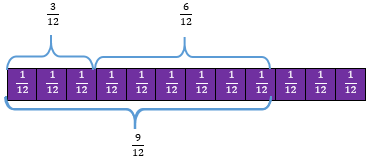
* It is important for students to draw these two models the same size. Once the models are created, students will then need to be able to make the all the pieces within each model the same size to be able to subtract. They then divide each piece of the model into fourths. They then divide each piece of the model into thirds. Now both models are divided in to 12 pieces and the subtraction problem can be represented as . It is important to note that the area of the models did not change. Just because the fraction changed, the value of the fraction did not change.



Now, students can subtract the same size pieces. So .



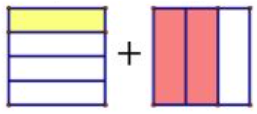
* Instruction includes concrete models and drawings that help solidify understanding that when adding and subtracting with unlike denominators, students are adding and subtracting pieces of the whole.
  + For example, the teacher emphasizes the use of area and number line models and presents expressions in numeral-word form to help understand that one over the denominator is the unit.
  + For example, “ are equal to how many ?” The denominator is 12 so one unit is equal to 1 twelfth.

****

Instructional Tasks

*Instructional Task 1 (MTR.2.1)*

Write an expression for the visual model below. Then find the sum.



*Instructional Task 2 (MTR.2.1)*

Use a visual fraction model to find the value of the expression .

*Instructional Task 3 (MTR.3.1)*

Find the value of the expression .

*Instructional Task 4 (MTR.3.1)*

Find the differences of and .

Instructional Items

*Instructional Item 1*

Find the sum .

*Instructional Item 2*

Find the difference .

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

### MA.5.FR.2.3

Benchmark

MA.5.FR.2.3 When multiplying a given number by a fraction less than 1 or a fraction greater than 1, predict and explain the relative size of the product to the given number without calculating.

Benchmark Clarifications:

*Clarification 1:* Instruction focuses on the connection to decimals, estimation and assessing the reasonableness of an answer.

Connecting Benchmarks/Horizontal Alignment

* MA.4.FR.2.4
* MA.5.NSO.2.4/2.5
* MA.5.GR.2.1

Terms from the K-12 Glossary

Vertical Alignment

|  |  |
| --- | --- |
| **Previous Benchmarks**   * MA.3.NSO.2.2 * MA.3.FR.1.2 | **Next Benchmarks**   * MA.6.NSO.2.2/2.3 |

Purpose and Instructional Strategies

The purpose of this benchmark is for students to examine how numbers change when multiplying by fractions *(MTR.2.1)*. Students already had experience with this idea when they multiplied a fraction by a whole number earlier in the Grade 4 Accelerated course (MA.4.FR.2.4). Work from this benchmark will help prepare students to multiply and divide fractions and decimals with procedural fluency in grade 6 (MA.6.NSO.2.2).

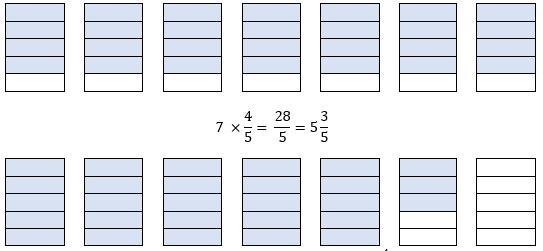
* It is important for students to have experiences examining:
  + when multiplying by a fraction greater than 1, the number increases;
  + when multiplying by a fraction equal to 1, the number stays the same; and
  + when multiplying by a fraction less the 1, the number decreases.
* Throughout instruction, encourage students to use models or drawings to assist them with a visual of the relative size. Models to consider when multiplying fractions to assist with finding relative size without calculating include, but are not limited to, area models (rectangles), linear models (fraction strips/bars and number lines) and set models (counters). Include examples with equivalent fractions and decimals *(MTR.2.1)*.
* Have students explain how they used the model or drawing to arrive at the solution and justify reasonableness of their answers *(MTR.4.1)*.

Common Misconceptions or Errors

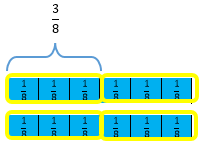
* Some may believe that multiplication always results in a larger number. This is why it is imperative to include models during instruction when multiplying fractions so students can see and experience the results and begin to make generalizations that are based on their understanding. Ultimately, allowing students to begin to understand that multiplying by a fraction less than one will result in a lesser product, but when multiplying by a fraction greater than one will result in a greater product.

Strategies to Support Tiered Instruction

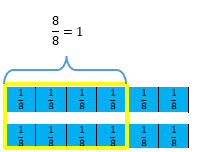
* Instruction includes opportunities to predict and explain the relative size of the product of a given number by a fraction less than one or a fraction greater than one. Students use models to check their prediction and solve. The teacher guides students to connect that multiplying a given number by a fraction less than one will result in a smaller number and that multiplying a given number by a fraction greater than one will result in a larger number.

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* + For example, the teacher displays the problem and asks students to predict if the product will be greater than, equal to, or less than 7 (it will be less than 7). Students use a visual model to represent the problem to determine . This is repeated with additional examples using fractions both greater than, equal to, and less than one.
* Instruction includes providing hands-on opportunities to predict and explain the relative size of the product of a given number by a fraction less than one or a fraction greater than one. Students use fraction strips/bars or counters to check their prediction and solve, connecting that multiplying a given number by a fraction less than one will result in a smaller number and that multiplying a given number by a fraction greater than one will result in a larger number.
* For example, the teacher displays the problem . Then, the teacher asks students to predict if the product will be greater than, equal to, or less than 4 (it will be less than 4).
  + Using fraction bars or fraction strips, the teacher models solving this problem with explicit instruction and guided questioning. Students explain how to use fraction bars or fraction strips as a model to solve this problem.

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* + This is repeated with additional examples using fractions both greater than, equal to, and less than one.

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Instructional Tasks

*Instructional Task 1 (MTR.3.1, MTR.4.1)*

Derrick is playing a computer game where he must multiply a number by a factor that increases the number’s size each time. Select all of the factors that he could multiply by to continue to increase the size of his number? Explain your thinking.

Instructional Items

*Instructional Item 1*

Which of the following expressions will have a product greater than 4?



*Instructional Item 2*

Fill in the blank. The product of the expression will be \_\_\_\_\_\_\_\_\_\_\_\_\_ 20.

* 1. less than
  2. equal to
  3. greater than
  4. half of

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

### MA.5.FR.2.4

Benchmark

MA.5.FR.2.4 Extend previous understanding of division to explore the division of a unit fraction by a whole number and a whole number by a unit fraction.

Benchmark Clarifications:

*Clarification 1:* Instruction includes the use of manipulatives, drawings or the properties of operations.

*Clarification 2:* Refer to Situations Involving Operations with Numbers (Appendix A).

Connecting Benchmarks/Horizontal Alignment

* MA.4.FR.2.4
* MA.5.NSO.2.2
* MA.5.AR.1.3

Terms from the K-12 Glossary

Vertical Alignment

|  |  |
| --- | --- |
| **Previous Benchmarks**   * MA.3.NSO.2.2 * MA.3.FR.1.2 | **Next Benchmarks**   * MA.6.NSO.2.2/2.3 |

Purpose and Instructional Strategies

The purpose of this benchmark is for students to experience division with whole number divisors and unit fraction dividends (fractions with a numerator of 1) and with unit fraction divisors and whole number dividends. This work prepares for division of fractions in grade 6 (MA.6.NSO.2.2) in the same way that MA.4.FR.2.4 prepared students for multiplication of fractions.

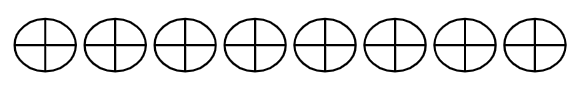
* Instruction should include the use of manipulatives, area models, number lines, and emphasizing the properties of operations (e.g., through fact families) for students to see the relationship between multiplication and division *(MTR.2.1)*.
* Throughout instruction, students should have practice with both types of division: a unit fraction that is divided by a non-zero whole number and a whole number that is divided by a unit fraction.
* Students should be exposed to all situation types for division (refer to Situations Involving Operations with Numbers (Appendix A)).
* The expectation of this benchmark is not for students to use an algorithm (e.g., multiplicative inverse) to divide by a fraction.
* Instruction includes students using equivalent fractions to simplify answers; however, putting answers in simplest form is not a priority.

Common Misconceptions or Errors

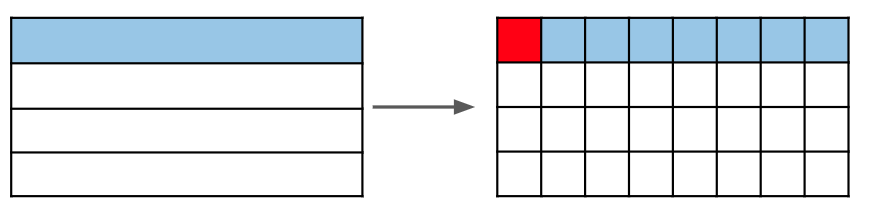
* Students may believe that division always results in a smaller number, which is true when dividing a fraction by a whole number, but not when dividing a whole number by a fraction. Using models will help students develop the understanding needed for computation with fractions.

Strategies to Support Tiered Instruction

* Instruction includes making the connection to models and tools previously used to understand division as equal groups or sharing. The teacher uses models to develop the understanding needed for computation with fractions.
* For example, can be shown using a model of 8 wholes divided into parts of size . The quotient would be the total number of pieces. The model below would show that .



* + For example, can be represented using the model below. One-fourth is divided into 8 equal parts, each part is of the whole.



* Instruction includes real-world situations to interact with the content. The teacher provides students with a division expression with a real-world context and provides items to represent the situation to allow connections to be made.
  + For example, the teacher provides students with the following situation: “The teacher brought in 8 brownies to split between the class. She cut the brownies into pieces of size so there would be enough for the whole class. How many pieces will there be?” The teacher provides students with images of eight brownies (or models to represent them) and has them divide or cut them into pieces to determine how many pieces they will have (32 pieces).
* For example,the teacher provides students with the following situation: “The teacher baked a pan of brownies. All but of the pan was eaten. She brought in the remaining and divided it into 8 equal pieces for her co-teachers. What fraction of the whole pan will each person get?”The teacher provides students with an image of a pan of brownies with left (or model to represent it). The students divide the portion into 8 equal pieces. The teacher then connects the remaining part of the brownies to the whole pan so that students can make the connection to the total number of the smaller pieces representing of the whole.

Instructional Tasks

*Instructional Task 1 (MTR.5.1, MTR.7.1)*

Part A. Emily has 2 feet of ribbon to make friendship bracelets. Use models and equations to answer the questions below.

1. How many friendship bracelets can she make if each bracelet uses 2 feet of ribbon?
2. How many friendship bracelets can she make if each bracelet uses 1 foot of ribbon?
3. How many friendship bracelets can she make if each bracelet uses 1 half foot of ribbon?
4. How many friendship bracelets can she make if each bracelet uses 1 third foot of ribbon?
5. How many friendship bracelets can she make if each bracelet uses 1 fifth foot of ribbon?

Part B. Do you see any patterns in the models and equations you have written? Explain.

Instructional Items

*Instructional Item 1*

What is the quotient of ?

*Instructional Item 2*

How many fourths are in 8 wholes?

1. 4
2. 8
3. 16
4. 32

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

## Algebraic Reasoning

**MA.4.AR.1** *Represent and solve problems involving the four operations with whole numbers and fractions.*

### MA.4.AR.1.1

Benchmark (Grade 4)

MA.4.AR.1.1 Solve real-world problems involving multiplication and division of whole numbers including problems in which remainders must be interpreted within the context.

*Example:* A group of 243 students is taking a field trip and traveling in vans. If each van can hold 8 students, then the group would need 31 vans for their field trip because 243 divided by 8 gives 30 with a remainder of 3.

Benchmark Clarifications:

*Clarification 1:* Problems involving multiplication include multiplicative comparisons. Refer to [Situations Involving Operations with Numbers (Appendix A)](#_Appendix_A:_Situations).

*Clarification 2:* Depending on the context, the solution of a division problem with a remainder may bethe whole number part of the quotient, the whole number part of the quotient with the remainder, the whole number part of the quotient plus 1, or the remainder.

*Clarification 3:* Multiplication is limited to products of up to 3 digits by 2 digits. Division is limited to up to 4 digits divided by 1 digit.

Benchmark (Grade 5)

MA.5.AR.1.1 Solve multi-step real-world problems involving any combination of the four operations with whole numbers, including problems in which remainders must be interpreted within the context.

Benchmark Clarifications:

*Clarification 1:* Depending on the context, the solution of a division problem with a remainder may bethe whole number part of the quotient, the whole number part of the quotient with the remainder, the whole number part of the quotient plus 1, or the remainder.

Connecting Benchmarks/Horizontal Alignment

* MA.5.NSO.2.1/2.2
* MA.5.FR.1.1
* MA.5.GR.3.3
* MA.5.GR.4.2
* MA.5.DP.1.2

Terms from the K-12 Glossary

* + Dividend
  + Divisor
  + Equation

Vertical Alignment

|  |  |
| --- | --- |
| **Previous Benchmarks**  MA.3.AR.1.2 | **Next Benchmarks**   * MA. 6.NSO.2.3 |

Purpose and Instructional Strategies

The purpose of this benchmark is for students to solve multistep word problems with whole numbers and whole-number answers involving any combination of the four operations. Work in this benchmark continues instruction from the Grade 3 Accelerated course where students interpreted remainders in division situations (MA.4.AR.1.1) *(MTR.7.1)*, and prepares students for solving multi-step word problems involving fractions and decimals in Grade 6 (MA.6.NSO.2.3).

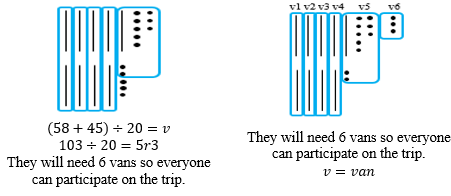
* To allow for an effective transition into algebraic concepts in Grade 6 (MA.6.AR.1.1), it is important for students to have opportunities to connect mathematical statements and number sentences or equations.
* During instruction, teachers can allow students an opportunity to practice with word problems that require multiplication or division which can be solved by using drawings and equations, especially as the students are making sense of the context within the problem *(MTR.5.1)*.
* Teachers can have students practice with representing an unknown number in a word problem with a variable by scaffolding from the use of only an unknown box.
* Offer word problems to students with the numbers covered up or replaced with symbols or icons and ensure to ask students to write the equation or the number sentence to show the problem type situation *(MTR.6.1)*.
* Interpreting number pairs on a coordinate graph can provide students opportunities to solve multi-step real-world problems with the four operations (MA.5.GR.4.2).

Common Misconceptions or Errors

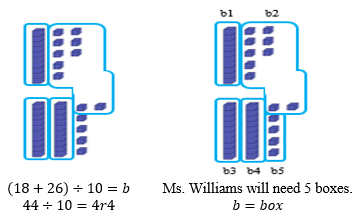
* Students may apply a procedure that results in remainders that are expressed as for ALL situations, even for those in which the result does not make sense.
  + For example, when a student is asked to solve the following problem: “There are 34 students in a class bowling tournament. They plan to have 3 students in each bowling lane. How many bowling lanes will they need so that everyone can participate?” the student response is “11 1 bowling lanes,” without any further understanding of how many bowling lanes are needed and how the students may be divided among the last 1 or 2 lanes. To assist students with this misconception, pose the question “What does the quotient mean?”
* It is essential to guide students in tackling real-world problems that may provide a quotient with a remainder, where interpretation may be necessary. Foster a mindset that extends beyond the literal content of the word problem. For example, prompt students to consider whether it is plausible to have half of a person/object or if the problem is seeking complete groups. Remind students that interpreting remainders is contingent upon the problem's context and the specific information the problem aims to reveal. Encourage critical thinking and application of mathematical concepts in practical scenarios.

Strategies to Support Tiered Instruction

|  |
| --- |
| * Instruction includes opportunities to engage in guided practice completing multi-step word problems with any combination of the four operations, including problems with remainders. Students use drawings and models to understand how to interpret the remainder in situations in which they will need to drop the remainder as their solution.   + For example, the teacher displays and reads the following problem aloud: “There are 58 fourth grade students and 45 fifth grade students going on a class field trip. They plan to have 20 students in each van. How many vans will they need so that everyone can participate?”Students use models or drawings to represent the   + problem and write an equation to represent the problem. The teacher uses guided questioning to encourage students to identify that they will need to add one to the quotient as their solution. If students state that they will need vans, the teacher refers to the models to prompt students that a sixth van is needed for the remaining three students. If students state that they will need 3 more vans since the remainder is 3, the teacher reminds students through guided questioning that the remainder of 3 represents 3 remaining students and only 1 more van is needed (i.e., “add 1 to the quotient”). This is repeated with similar multistep real-world problems, asking students to explain what the quotient means in problems involving remainders. |

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* Instruction includes opportunities to engage in practice with explicit instruction completing multi-step word problems with any combination of the four operations, including problems with remainders. Students use manipulatives to understand how to interpret the remainder in situations in which they will need to drop the remainder as their solution.
  + For example, the teacher displays and reads the following problem aloud: “There are 18 red markers and 26 black markers on the art table. Ms. Williams is cleaning up and can put 10 markers in each box. How many boxes will she need so all the markers will be put into box?” The teacher uses manipulatives (e.g., base ten blocks) to represent the problem, having students write an equation to represent the problem. The teacher uses guided questioning to encourage students to identify that they will need to add 1 to the quotient as their solution. If students state that she will need 4 boxes, the teacher refers to the models to prompt students that a fifth box is needed for the remaining four markers. If students state that they will need 4 more boxes since the remainder is 4, the teacher reminds students through guided questioning that the remainder of 4 represents 4 remaining markers and only 1 more box is needed (i.e., “add 1 to the quotient”). This is repeated with similar multistep real-world problems, asking students to explain what the quotient means in problems involving remainders.



Instructional Tasks

*Instructional Task 1 (MTR.6.1)*

There are 128 girls in the Girl Scouts Troop 1653 and 154 girls in the Girl Scouts Troop 1764. Both Troops are going on a camping trip. Each bus can hold 36 girls. How many buses are needed to get all the girls to the camping site?

*Instructional Task 2*

Agusta had $2,385. She spent $729 on a new computer. She wants to spend the rest of her money on different computer accessories. Each accessory cost $65. What is the maximum number of accessories Augusta can buy?

* + 1. 25 accessories
    2. 26 accessories
    3. 27 accessories
    4. 28 accessories

Instructional Items

*Instructional Item 1*

A shoe store orders 17 cases each containing 142 pairs of sneakers and 12 cases each containing 89 pairs of sandals. How many more pairs of sneakers did the store order?

*Instructional Item 2 (MTR.3.1)*

Stanley saved $342 last month and $780 this month. He wants to buy as many chairs to place in the auditorium at school as possible. Each chair costs $11. How many chairs can Stanley buy?

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

### MA.4.AR.1.3 & MA.5.AR.1.2

Benchmark (Grade 4)

MA.4.AR.1.3 Solve real-world problems involving multiplication of a fraction by a whole number or a whole number by a fraction.

*Example:* Ken is filling his garden containers with a cup that holds pounds of soil. If he uses 8 cups to fill his garden containers, how many pounds of soil did Ken use?

Benchmark Clarifications:

*Clarification 1:* Problems include creating real-world situations based on an equation or representing a real-world problem with a visual model or equation.

*Clarification 2:* Fractions within problems must reference the same whole.

*Clarification 3:* Within this benchmark, the expectation is not to simplify or use lowest terms.

*Clarification 4:* Fractions limited to fractions less than one with denominators of 2, 3, 4, 5, 6, 8, 10, 12, 16 and 100.

Benchmark (Grade 5)

MA.5.AR.1.2 Solve real-world problems involving the addition, subtraction or multiplication of fractions, including mixed numbers and fractions greater than 1.

*Example:* Shanice had a sleepover, and her mom is making French toast in the morning. If her mom had loaves of bread and used loaves for the French toast, how much bread does she have left?

Benchmark Clarifications:

*Clarification 1:* Instruction includes the use of visual models and equations to represent the problem.

Connecting Benchmarks/Horizontal Alignment

* MA.4.FR.2.4
* MA.4.M.1.2
* MA.4.DP.1.3
* MA.5.AR.1.2/1.3

Terms from the K-12 Glossary

* Equation
* Expression
* Whole Number

Vertical Alignment

|  |  |
| --- | --- |
| **Previous Benchmarks**   * MA.3.FR.1.2 | **Next Benchmarks**   * MA. 6.NSO.2.3 |

Purpose and Instructional Strategies

The purpose of this benchmark is to continue the work from the Grade 3 Accelerated course (MA.4.AR.1.2) where students began solving real-world problems with fractions and prepares them for Grade 6 (MA.6.NSO.2.3) where they will solve real-world problems with fractions using all four operations with fractions *(MTR.7.1)*.

* Students need to develop an understanding that when adding or subtracting fractions, the fractions must refer to the same whole.
* During instruction, teachers should provide opportunities for students to practice solving problems using models or drawings to add, subtract or multiply with fractions. Begin with students modeling with whole numbers, have them explain how they used the model or drawing to arrive at the solution, then scaffold using the same methodology using fraction models. Instruction allows students to build understanding fractions through modeling and the use of manipulatives.
* Students are encouraged to express problem-solving solutions through various means, such as objects, illustrations, tables, graphs, and equations (MTR.2.1). When tackling fraction-related challenges, it is advisable to employ diverse models, including but not confined to area models (rectangles), linear models (fraction strips/bars and number lines), and set models (counters).There should be student progression from solving problems with tangible objects and illustrations to the utilization of algorithms and equations. Students should articulate the relationships between different concepts and representations, while also selecting the most suitable representation based on the specific context or purpose of the question.
* Please note that it is not expected for students to always find least common multiples or make fractions greater than 1 into mixed numbers, but it is expected that students know and understand equivalent fractions, including naming fractions greater than 1 as mixed numbers to add, subtract or multiply.
* It is important that teachers have students rename the fractions with a common denominator when solving addition and subtraction fraction problems in lieu of the “butterfly” method (or other shortcut/mnemonic) to ensure students build a complete conceptual understanding of what makes solving addition and subtraction of fractions problems true.

Common Misconceptions or Errors

* When solving real-world problems, students can often confuse contexts that require subtraction and multiplication of fractions.
  + For example, “Mark has yards of rope and he gives half of the rope to a friend. How much rope does Mark have left?” expects students to find of , or multiply to find the product that represents how much is given to the friend. On the other hand, “Mark has yards of rope and gives yard of rope to a friend. How much rope does Mark have left?” expects students to take yard from yard, or subtract to find the difference. Encourage students to look for the units in the problem (e.g., yard versus of the whole rope) to determine the appropriate operation.
* Students may believe that multiplication always results in a larger number. Using models when multiplying with fractions will enable students to generalize about multiplication algorithms that are based on conceptual understanding (*MTR.5.1*). Encourage students to use their number sense when determining the size of a product. The teacher can ask students what they know about multiplying fractions to determine how the numbers in the problem are related.
* Students can have difficulty with word problems when determining which operation to use, and the stress of working with fractions makes this happen more often. Students need to apply what they already know about operations with fractions to create action plans to solve multi-step real-world problems involving fractions.

For example, “Mark has yards of rope and he gives a third of the rope to a friend. How much rope does Mark have left?” expects students to first find of or multiply , and then to find the difference to find how much Mark has left. On the other hand, “Mark has yards of rope and gives yard of rope to a friend. How much rope does Mark have left?” only requires finding the difference .

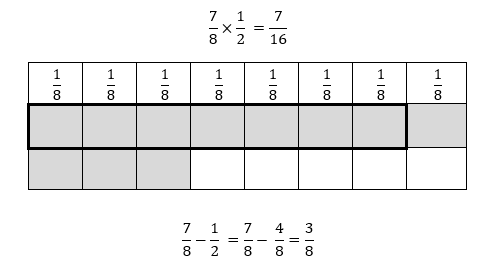
* + With guiding questions, ask students what operation will be needed to solve the problem. Prompt students to tell you what the first action step will be before they solve. If students struggle to find the first step of the problem, encourage them to create a visual model while rereading

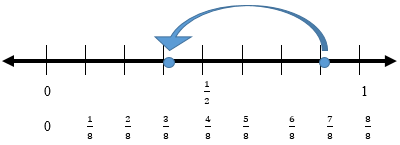
Strategies to Support Tiered Instruction

Instruction includes opportunities to identify the appropriate operation to use in a real-world problem that requires addition, subtraction, or multiplication of fractions. The teacher guides students to identify the units in the problem for clarification on which operation is appropriate.

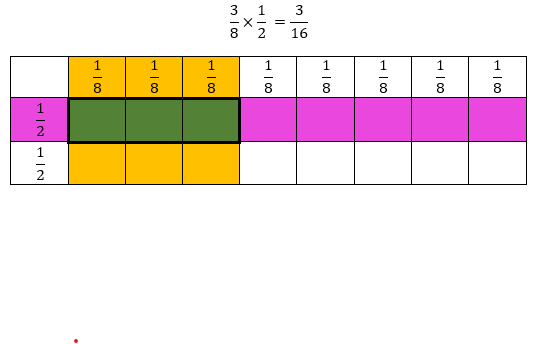
* + For example, the teacher displays and reads the following two problems:
    - “Gina has of a bar of chocolate left and gives half of what she has to her friend Sarah. How much of a whole chocolate bar does she have left?”
    - “Gina has of a bar of chocolate left and she gives of the original bar of chocolate to her friend Sarah. How much of her chocolate bar does she have left?” (See illustration below)

The teacher uses questioning and prompting to have students identify what operations must be used to solve each problem. The teacher asks students to share what they notice about each problem (e.g., the similarities and the differences), placing emphasis on the units (e.g., “half of the amount of chocolate that Gina has in the first problem vs. of the whole chocolate bar*”* in the second problem). The teacher guides students to identify that in the first problem, they will need to multiply and in the second problem, they will need to subtract to solve. Students solve using models.

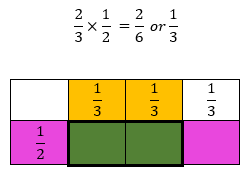


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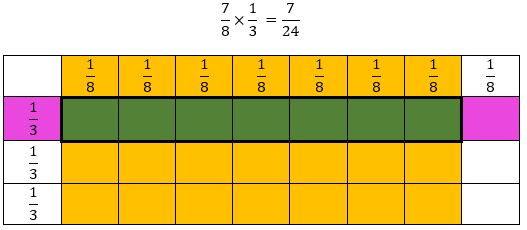
* + For example, the teacher displays and reads the following problem: “Tia has yards of ribbon and she gives half of the ribbon to a friend. How much ribbon does Tia have left?” The teacher uses questioning and prompting to have students identify what operation must be used to solve the problem. The teacher asks students, “Did Tia give half of the ribbon or half a yard of ribbon to her friend?” Emphasis is placed on the units (e.g., half of the whole ribbon vs. yard of ribbon) while guiding students to identify that they will need to multiply to solve. Students solve using the area model and counters. The cells with both color counters indicate the numerator in the solution. This is repeated with similar word problems, using frequent guiding questions to support student understanding.



* Instruction includes opportunities to use models when solving problems that involve multiplication of fractions to increase understanding that multiplication does not always result in a larger number. The use of models when multiplying with fractions will enable students to generalize about multiplication algorithms that are based on conceptual understanding.
  + For example, the teacher displays and reads aloud the following problem: “Rosalind spent of an hour helping in the garden. Her sister spent the amount of time as Rosalind did helping in the garden. How much time did Rosalind’s sister spend helping in the garden?” Students solve the problem using an area model. The teacher uses questioning to help students draw a model to represent the problem. This is repeated with similar word problems involving multiplication of fractions.

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* + For example, the teacher displays and reads aloud the following problem: “Astrid spent of an hour reading her book. Elliot spent the amount of time as Astrid did reading. How much time did Elliot spend reading?” Students solve using the area model and counters. The cells with both color counters indicate the numerator in the solution. The teacher uses questioning to help students draw a model to represent the problem. This is repeated with similar word problems involving multiplication of fractions, using frequent guiding questions to support student understanding.



Instructional Tasks

*Instructional Task 1 ((MTR.7.1)*

Rachel wants to bake her two favorite brownie recipes. One recipe needs cups of flour and the other recipe needs cups of flour. How much flour does Rachel need to bake her two favorite brownie recipes?

*Instructional Task 2 (MTR.7.1)*

Shawn finished a 100 meter race in of one minute. The winner of the race finished in of Shawn’s time. How long did it take for the winner of the race to finish?

*Instructional Task 3*

Michelle paints a wall that is 6 feet high and 62feet long. How many square feet does he paint?

* + 1. 372square feet
    2. 392square feet
    3. 392square feet
    4. 393square feet

Instructional Items

*Instructional Item 1*

Monica has 2 cups of berries. She uses cups of berries to make a smoothie. She then uses cup for a fruit salad. After she makes her smoothie and fruit salad, how much of the berries will Monica have left?

*Instructional Item 2*

Megan has a collection of 80 stickers, and of them feature cars, while of the stickers have animals. The remaining stickers display various types of shapes. How many stickers in Megan’s collection feature shapes?

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

***MA.5.AR.1*** *Solve problems involving the four operations with whole numbers and fractions.*

### MA.5.AR.1.1

### See Benchmark MA.4.AR.1.1 (insert hyperlink here)

### MA.5.AR.1.2

### See Benchmark MA.4.AR.1.3 (insert hyperlink here)

### MA.5.AR.1.3

Benchmark

MA.5.AR.1.3 Solve real-world problems involving division of a unit fraction by a whole number and a whole number by a unit fraction.

*Example:* A property has a total of acre and needs to be divided equally among 3 sisters. Each sister will receive of an acre.

*Example:* Kiki has 10 candy bars and plans to give of a candy bar to her classmates at school. How many classmates will receive a piece of a candy bar?

Benchmark Clarifications:

*Clarification 1:* Instruction includes the use of visual models and equations to represent the problem.

Connecting Benchmarks/Horizontal Alignment

* MA.4.AR.1.3
* MA.5.NSO.2.2
* MA.5.FR.2.4

Terms from the K-12 Glossary

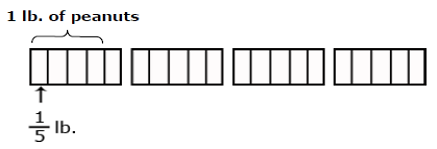
Vertical Alignment

|  |  |
| --- | --- |
| **Previous Benchmarks**   * MA.4.AR.1.3 | **Next Benchmarks**   * MA.6.NSO.2.3 |

Purpose and Instructional Strategies

The purpose of this benchmark is to connect division of fraction concepts to real-world scenarios *(MTR.7.1)*. This work builds on the multiplication of fractions by whole numbers from earlier in the Grade 4 Acceleration course (MA.4.AR.1.3) and prepares them for Grade 6 (MA.6.NSO.2.3) where they will solve real-world fraction problems using all four operations with fractions (*MTR.7.1*). Additionally, students divide a unit fraction by a non-zero whole number (MA.5.FR.2.4) using division concepts learned in (5.NSO.2.2).

* During instruction, it is important for students to have opportunities to extend their understanding of the meaning of fractions, how many unit fractions are in a whole, and their understanding of division of fractions as involving equal groups or shares and the number of objects in each.
* Students should use visual fraction models and reasoning to solve word problems involving division of fractions.
  + For example, to assist students with solving the problem, “The elephant eats 4 pounds of peanuts a day. His trainer gives him of a pound at a time. How many times a day does the elephant eat peanuts?” use the following diagram to show how can be visualized to assist students with solving



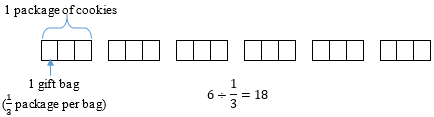
* The expectation of this benchmark is not for students to use a specific algorithm multiplicative inverse for dividing fractions. Instead, it is encouraged for students to enhance their understanding by drawing and labeling diagrams. Teachers should present real-world scenarios (MTR.7.1) where it is inherent for students to divide a unit fraction by a non-zero whole number or a whole number by a unit fraction.
* Instruction includes allowing students to visualize and practice using patterns and structure to divide unit fractions by non-zero whole numbers (*MTR.5.1*).
* Instruction includes students using equivalent fractions to simplify answers; however, putting answers in simplest form is not a priority.

Common Misconceptions or Errors

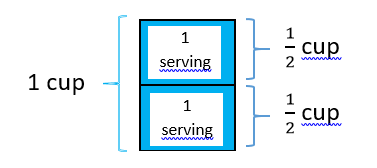
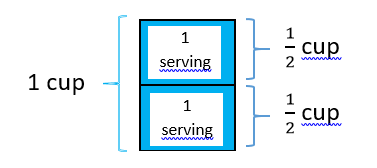
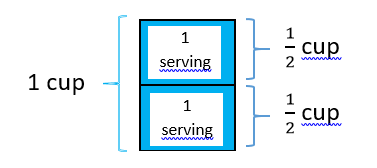
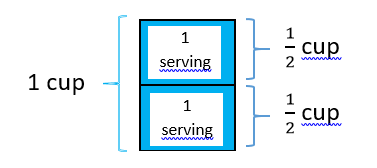
* Students may believe that division always results in a smaller number, which is true when dividing a fraction by a whole number, but not when dividing a whole number by a fraction. Using models will help students develop the understanding needed for computation with fractions.

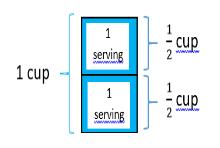
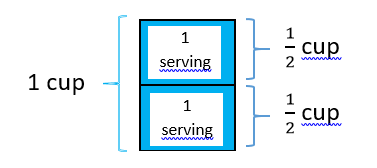
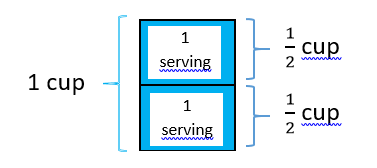
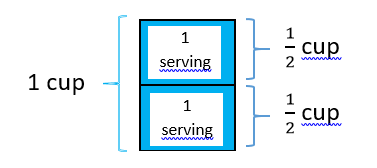
Strategies to Support Tiered Instruction

* Instruction includes opportunities to engage in teacher-directed practice using visual representations to solve real-world problems involving division of a unit fraction by a whole number or a whole number by a unit fraction. The teacher directs students on how to use models or equations based on real-world situations. Through questioning, the teacher guides students to explain what each fractional portion represents in the problems used during instruction and practice.
  + For example, the teacher displays and reads aloud the following problem: “Julio has 6 packages of cookies. He is making gift bags for people at school. Each bag will contain of a package of cookies. How many gift bags can he make?”

****Using models, the teacher solves the problem with guided questioning, having students explain how to use models to solve this question. The teacher guides students to create an equation to represent the problem. This is repeated with multiple real-world examples that involve division of a unit fraction by a whole number or a whole number by a unit fraction. If a student struggles to draw a picture of the problem, guide them along by asking what item in the question is being divided, and into how many/what size parts. Encourage and remind students to use pictures or objects.

* Teacher provides opportunities to use hands-on models and manipulatives to solve real-world problems involving division of a unit fraction by a whole number or a whole number by a unit fraction. Students explain how each model represents the real-world situation. The teacher directs students how to use models or equations based on real-world examples and through questioning guide students to explain what each fractional portion represents in the problems used during instruction and practice.
  + For example, the teacher displays and reads aloud the following problem: “Shelton made some lemonade. The pitcher of lemonade holds 8 cups. If each of the glasses that he uses can hold cup, how many servings of lemonade can he share?” Using fraction bars or fraction strips, the teacher models solving the problem with explicit instruction and guided questioning. Students explain how to use fraction bars or fraction strips as a model to solve this question and use an equation to represent the problem. This is repeated with multiple real-world problems that involve multiplication of a whole number by a fraction or a fraction by a whole number.





Instructional Tasks

*Instructional Task 1 ((MTR.6.1, MTR.7.1)*

Sonya has gallon of chocolate chip ice cream. She wants to share her ice cream with 6 friends. How much ice cream will each friend get?

*Instructional Task 2 (MTR.6.1, MTR.7.1)*

A string that is 8 feet long is cut into pieces that are feet long. How many pieces are there?

Instructional Items

*Instructional Item 1*

Betty has 12 sheets of tissue paper to add to her holiday gift bags. Each gift bag needs sheet of tissue paper. How many holiday gift bags can Betty fill?

*Instructional Item 2*

Mr. Smith has a rectangular table with an area of 15 square feet and another square table with an area of 9 square feet. He plans to cover both tables with tiles, where each tile has an area of square foot. How many tiles does he need to completely cover both tables without any gaps or overlaps?

*Instructional Item 3 (MTR.5.1, MTR.6.1)*

Use operation symbols to make the equation true.

Equation for instructional item 3 above.

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

**MA.5.AR.2***Demonstrate an understanding of equality, the order of operations and equivalent numerical expressions.*

### MA.5.AR.2.1

Benchmark

MA.5.AR.2.1 Translate written real-world and mathematical descriptions into numerical expressions and numerical expressions into written mathematical descriptions.

*Example:* The expression in word form is plus the quantity times .

Benchmark Clarifications:

*Clarification 1:* Expressions are limited to any combination of arithmetic operations, including parentheses, with whole numbers, decimals and fractions.

*Clarification 2:* Within this benchmark, the expectation is not to include exponents or nested grouping symbols.

Connecting Benchmarks/Horizontal Alignment

* MA.5.NSO.1
* MA.5.NSO.2.1, MA.5.NSO.2.2
* MA.5.NSO.2.3
* MA.5.AR.3.1
* MA.5.M.1.1

Terms from the K-12 Glossary

* Expression

Vertical Alignment

|  |  |
| --- | --- |
| **Previous Benchmarks**   * MA.4.AR.2.2 | **Next Benchmarks**   * MA.6.AR.1.1 |

Purpose and Instructional Strategies

* The purpose of this benchmark is for students to translate between numerical and written mathematical expressions. This builds from previous work where students wrote equations with unknowns in any position of the equation in Grade 4 (MA.4.AR.2.2). Algebraic expressions are a major theme in Grade 6 starting with MA.6.AR.1.1. Students continue to deepen their understanding of operations with whole numbers (MA.5.NSO.2.1/2.2).
* During instruction, teachers should model how to translate numerical expressions into words using correct vocabulary. This includes naming fractions and decimals correctly. Students should use diverse vocabulary to describe expressions.
  + For example, in the expression could be read in multiple ways to show its operations. Students should explore them and find connections between their meanings *(MTR.1.1, MTR.3.1, MTR.4.1, MTR.5.1)*.
    - *4 and five tenths plus the quantity 3 times 2*
    - *4 and 5 tenths plus the product of 3 and 2*
    - *The sum of 4 and 5 tenths and the quantity 3 times 2*
    - *The sum of 4 and 5 tenths and the product of 3 and 2*
* The expectation of this benchmark is to not use exponents or nested grouping symbols. Nested grouping symbols, such as braces and brackets, refer to grouping symbols within one another in an expression, like in . Explain to students that parentheses make the expression clearer, but they are not necessary when multiplication comes before addition, as seen in the example.
* Instruction of this benchmark helps students understand the order of operations, the expectation of MA.5.AR.2.2.

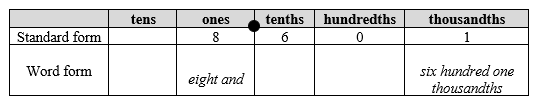
Teachers can encourage and motivate students to utilize properties to rewrite expressions. Instruction includes a review of the distributive, associative, and commutative properties of addition and multiplication. Reinforce the concept that the distributive property involves the distribution of multiplication over addition, while the commutative property entails a change in the order of the terms

Common Misconceptions or Errors

* Students can misrepresent decimal and fraction numbers in words. This benchmark helps students practice naming numbers according to place value.
* Some students can confuse the difference between what is expected in the expressions and Students need practice naming the former as multiplication (e.g., 5 times the sum of 9 and 3) and understanding that in that expression, both and are factors.
* Students may have confusion with parentheses. Misplacement or omission of parentheses can lead to incorrect interpretations of expressions. Students might forget to use parentheses where necessary or include unnecessary ones, altering the meaning of the expression.
* Additional misinterpretations include misunderstanding the order of operations, where a student mistakenly performs an operation over another (e.g. addition before multiplication with no parentheses present).

Strategies to Support Tiered Instruction

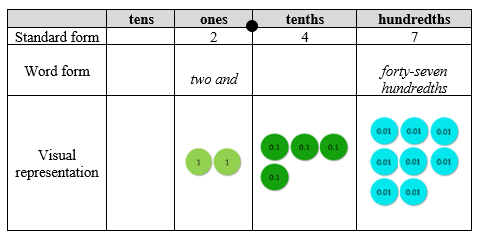
* Instruction includes opportunities to name fractions and decimals correctly according to place value. The teacher provides students a place value chart to support correctly naming decimals. Students use appropriate terminology for naming fractions.
  + For example, write 8.601 in standard form and word form in a place value chart.



* + For example, students write 10.36 in standard form and word form in a place value chart.



* + For example, students write 2.47 in standard form and word form in a place value chart using place value disks. Place value disks can be used as a visual representation in addition to the place value chart.



* + For example, students write in word form (*five twelfths*).
  + For example, students write in word form (*two and seven eighths*).

This is repeated with additional fractions and decimals.

* Instruction includes opportunities to correctly translate numerical expressions into words using appropriate vocabulary.
  + For example, the teacher has students read aloud the following expression and write in word form. Next, the teacher models one way of reading aloud and has students provide alternate ways while using guiding questioning to facilitate the conversation about the multiple ways the expression can be read aloud to show its operations (*MTR.2.1*)
    - Eighteen and forty-nine hundredths minus the quotient of twenty-seven divided by three.
    - 18 and 49 hundredths minus the quantity 27 divided by 3.
    - The difference between 18 and 49 hundredths and the quotient of 27 divided by 3.
    - The difference between 18 and 49 hundredths and the quantity 27 divided by 3.

Instructional Tasks

*Instructional Task 1 (MTR.4.1)*

Nadia sees the numerical expression . She translates the expression as, “.”

Part A: Is her translation correct? Explain.

Part B: Evaluate the expression.

*Instructional Task 2 (MTR.3.1)*

Translate the written mathematical description below into a numerical expression:

*Divide the difference between 20 and 5 by the sum of 4 and 1.*

*Instructional Task 3*

Melissa spends time each evening designing bracelets for her friends. The number of minutes she spends can be represented by the variable . Use the variable to complete the following tasks.

1. Write an algebraic expression to represent the number of minutes Melissa’s friend Gabriella spends designing bracelets in an evening if she designs 45 minutes more that evening than Melissa.
2. Write an algebraic expression to represent the number of minutes Melissa will spend designing bracelets at home in 9 weeks.

*Instructional Task 4*

Below is a model that represents .

addition model

1. Draw a model that represents .
2. How much times greater is the value of than ?

Instructional Items

*Instructional Item 1*

Translate the numerical expression below into a written mathematical description.

*Instructional Item 2*

Translate the written mathematical description into a numerical expression.

*“One half the difference of 6 and 8 hundredths and 2”*

*Instructional Item 3*

Which numerical expression is equivalent to the written expression?

4 times the quantity 97 plus 156

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

### MA.5.AR.2.2

Benchmark

MA.5.AR.2.2 Evaluate multi-step numerical expressions using order of operations.

Example: Patti says the expression is equivalent to because she works each operation from left to right. Gladys says the expression is equivalent to because first multiplies then divides into . David says that Patti is correctly using order of operations and suggests that if parentheses were added, it would give more clarity.

Benchmark Clarifications:

*Clarification 1:* Multi-step expressions are limited to any combination of arithmetic operations, including parentheses, with whole numbers, decimals and fractions.

*Clarification 2:* Within this benchmark, the expectation is not to include exponents or nested grouping symbols.

*Clarification 3:* Decimals are limited to hundredths. Expressions cannot include division of a fraction by a fraction.

Connecting Benchmarks/Horizontal Alignment

* MA.5.NSO.1
* MA.5.NSO.2.3 MA.5.NSO.2.4, MA.5.NSO.2.5
* MA.5.FR.1.1
* MA.5.FR.2.1

Terms from the K-12 Glossary

* Expression

Vertical Alignment

|  |  |
| --- | --- |
| **Previous Benchmarks**   * MA.4.AR.2.1, MA.4.AR.2.2 | **Next Benchmarks**   * MA.6.NSO.2.3 * MA.6.AR.1.3 |

Purpose and Instructional Strategies

The purpose of this benchmark is for students to use the order of operations to evaluate numerical expressions. In Grade 4, students had experience with numerical expressions involving all four operations (MA.4.AR.2.1/2.2), but the focus was not on order of operations. Students evaluate numerical expressions using whole-number computation skills, including multiplying and dividing multi-digit whole numbers (MA.5.NSO.2.1 and MA.5.NSO.2.2). In Grade 6, students will be evaluating algebraic expressions using substitution and these expressions can include negative numbers (MA.6.AR.1.3).

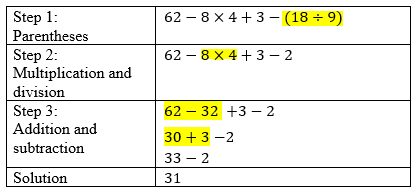
* Begin instruction by exposing student to expressions that have two operations without any grouping symbols, before introducing expressions with multiple operations. Use the same digits, with the operations in a different order, and have students evaluate the expressions, then discuss why the value of the expression is different.
  + For example, have students evaluate and .
* In Grade 5, students should learn to first work to simplify within any parentheses if present in the expression. Within the parentheses, the order of operations is followed. Next, while reading left to right, perform any multiplication and division in the order in which it appears. Finally, while reading from left to right, perform addition and subtraction in the order in which it appears.
* During instruction, students should be expected to explain how they used the order of operations to evaluate expressions and share with others. To address misconceptions around the order of operations, instruction should include reasoning and error analysis tasks for students to complete *(MTR.3.1, MTR.4.1, MTR.5.1)*.

Common Misconceptions or Errors

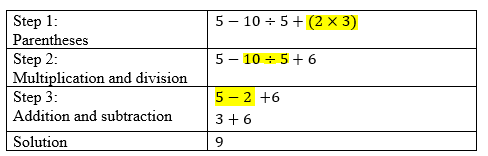
* When students learn mnemonics like PEMDAS to perform the order of operations, they may confuse that multiplication must always be performed before division, and likewise addition before subtraction. Students should have experience solving expressions with multiple instances of procedural operations and their inverse, such as addition and subtraction, so they learn how to solve them left to right.
  + For example, when given the expression , guide students to follow the order of operations and simplify the parentheses first. Next, students should recognize that subtraction and addition are the remaining operations, and the expression should be solved reading left to right.
* Students may understand the order in which to perform operations, but they may have difficulty keeping track of the numbers they have already operated with.

Strategies to Support Tiered Instruction

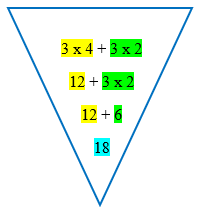
* Instruction includes opportunities to solve expressions with multiple instances of procedural operations and their inverse, explicitly teaching the order of operations with an emphasis on the left to right order to solving multiplication and division, and addition and subtraction. Students use models or drawings as they solve.
  + For example, the teacher displays the following problem: . The teacher reviews the order of operations, reminding students that they must work to simplify within the parentheses first. The teacher then prompts students to multiply and divide from left to right next. Then, students are prompted to add and subtract from left to right and reminded that adding and subtracting fall within the same step. So, they will need to subtract to get 30 and then add . The teacher repeats with additional expressions containing multiplication, division, addition, and subtraction in a variety of orders.



* Instruction includes manipulatives to practice solving expressions with multiple instances of procedural operations and their inverse, such as addition and subtraction, so they learn how to solve them left to right. Instruction also includes explicitly teaching the order of operations with an emphasis on the left to right order to solving multiplication and division, and addition and subtraction. Students use manipulatives as they solve.
  + For example, display the following problem: . The teacher reviews the order of operations, reminding students that they must work to simplify within the parentheses first. The teacher prompts students to multiply and divide from left to right next. Then, prompts students to add and subtract from left to right. Finally, the teacher reminds students that adding and subtracting falls within the same step, so they will need to subtract before they add . This is repeated with additional expressions containing multiplication, division, addition, and subtraction in a variety of orders.



* Instruction includes students using highlighters to keep track of the order of operations and the steps they have completed as they evaluate an expression, as shown above in the table. Another way to keep track of the steps is to have students write the expression in a triangle as seen below and color code each step and its value. Students make sure they have completed all the calculations by tracking each part of the expression down to the solution.



Instructional Tasks

*Instructional Task 1 (MTR.4.1)*

The two equations below are very similar. Are both equations true? Why or why not?

Equation One:

Equation Two:

*Instructional Task 2 (MTR.5.1)*

Part A. Insert one set of parentheses around two numbers in the expression below. Then evaluate the expression.

Part B. Now insert one set of parentheses around a different pair of numbers. Then evaluate this expression.

*Instructional Task 3*

Solve the following expression.

*Instructional Task 4*

Evaluate the following numerical expressions and determine if the parentheses can be removed without changing the value of the expression. Explain your thinking.

Instructional Items

*Instructional Item 1*

What is the value of the numerical expression below:

*Instructional Item 2*

A numerical expression is evaluated as shown.

In which step does the first mistake appear?

1. Step 1:
2. Step 2:
3. Step 3:
4. Step 4: 7

Instructional Item 3

Evaluate.

a.

b.

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

### MA.5.AR.2.3

Benchmark

MA.5.AR.2.3 Determine and explain whether an equation involving any of the four operations is true or false.

*Example:* The equation can be determined to be true because the expression on both sides of the equal sign are equivalent to .

Benchmark Clarifications:

*Clarification 1:* Problem types include equations that include parenthesis but not nested parentheses.

*Clarification 2:* Instruction focuses on the connection between properties of equality and order of operations.

Connecting Benchmarks/Horizontal Alignment

* MA.5.NSO.1.2/1.3
* MA.5.NSO.2.1, MA.5.NSO.2.3, MA.5.NSO.2.5
* MA.5.FR.2.4
* MA.5.AR.2.1, MA.5.AR.2.2
* MA.5.NSO.2.3, MA.5.NSO.2.5

Terms from the K-12 Glossary

* Equal Sign
* Equation
* Expression

Vertical Alignment

|  |  |
| --- | --- |
| **Previous Benchmarks**   * MA.4.AR.2.1 | **Next Benchmarks**   * MA.6.AR.2.1 |

Purpose and Instructional Strategies

The purpose of this benchmark is to determine if students can connect their understanding of using the four operations reliably or fluently *(MTR.3.1)* to the concept of the meaning of the equal sign. Students have evaluated whether equations are true or false since Grade 2. In Grade 5, additional expectations include non-whole numbers and parentheses. In Grade 6, students extend this work to involve negative numbers and inequalities (MA.6.AR.2.1).

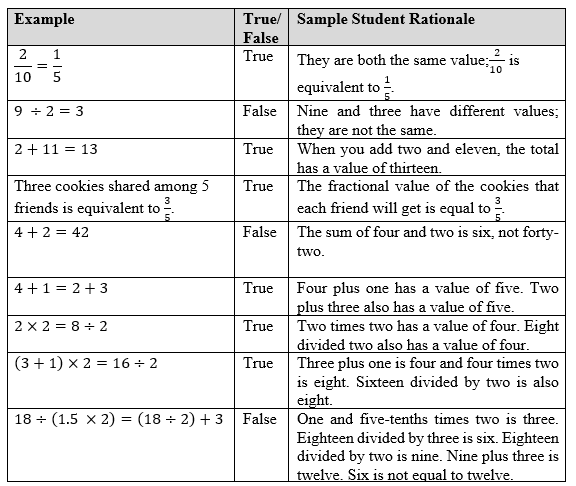
* Students will use their understanding of order of operations (MA.5.AR.2.2) to simplify expressions on each side of an equation *(MTR.5.1)*.
* Students will determine if the expression on the left of the equal sign is equivalent to the expression to the right of the equal sign. If these expressions are equivalent, then the equation is true.
* Students may use comparative relational thinking, instead of solving, in order to determine if the equation is true or false *(MTR.2.1, MTR.3.1, MTR.5.1)*.

Common Misconceptions or Errors

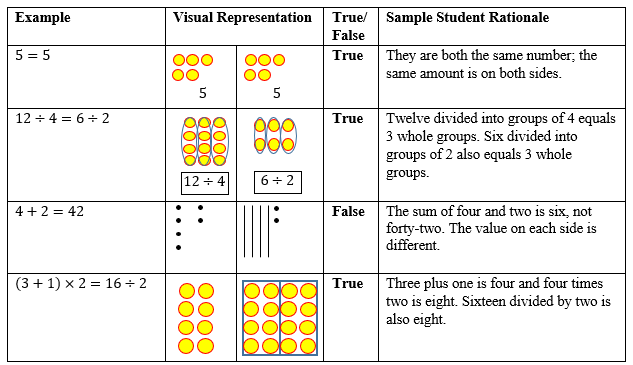
* Some students may not understand that the equal sign is a relational symbol showing expressions on both sides that are the same. While justifying whether equations are true or false, students should explain what makes the equation true.

Strategies to Support Tiered Instruction

* Instruction includes opportunities to explore the meaning of the equal sign. The teacher provides explicit clarification that the equal sign means “the same as” rather than “the answer is” along with multiple examples for students to evaluate equations as true or false using the four operations with the answers on both the left and right side of the equation. Instruction begins by using single numbers on either side of the equal sign to build understanding using the same equations written in different ways to reinforce the concept.
  + For example, the teacher shows the following equations, asking students if they are true or false statements. Students explain why each equation is true or false. This is repeated with additional true and false equations using the four operations.



* + For example, the teacher shows the following equations having students use counters, drawings, or base-ten blocks on a t-chart to represent the equation. The teacher asks students if they are true or false statements and to explain what makes equations true. This is repeated with additional true and false equations using the four operations.



* Instruction includes opportunities for students to organize and structure their algebraic tasks. Teachers should encourage students to label and annotate expressions on both sides of the equation prior to solving. This structured approach enables students to identify occasions where they can apply the properties of operations and the order of operations for problem-solving. Encourage students to contemplate the operations required to solve the expressions on both sides of the equation (*MTR.1.1*).

Instructional Tasks

*Instructional Task 1 (MTR.2.1)*

Using the numbers below, create an equation that is true.

, , , , 8

*Instructional Task 2*

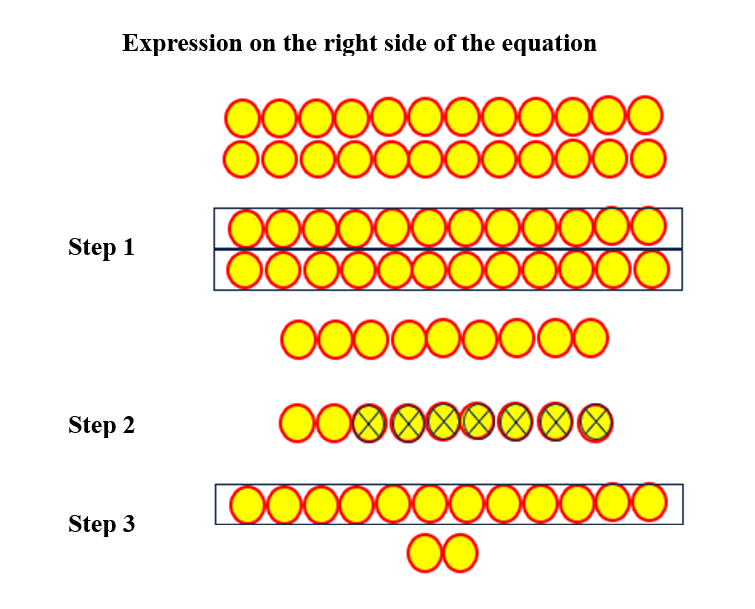
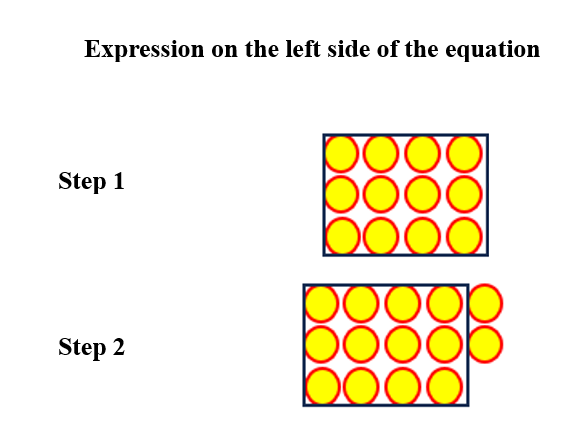
Part A. Determine if the following equation is true or false.

Part B. Determine if the models and steps shown correctly represent the expressions on both sides of the equation in Part A.

*Instructional Task 3*

Use each operation symbol once in the equation below to make it true.

9 \_\_\_ (7\_\_\_4) \_\_\_6\_\_\_2=34



Instructional Items

*Instructional Item 1*

Which best explains the equation below?

1. This equation is true because both sides of the equation are equal to 4.8.
2. This equation is true because both sides of the equation are equal to 10.8.
3. This equation is false because both sides of the equation are equal to 4.8.
4. This equation is false because both sides of the equation are unequal.

*Instructional Item 2*

Is the below equation true or false? Explain how you know.

*Instructional Item 3*

For what value of *x* is the below equation true? Explain.

1. The value of *x* is 2.5 because it makes both expressions equal 13.5.
2. The value of *x* is 13.5 because it is found when following the order of operations.
3. The value of *x* is 16 because it is value on the right side of the equal sign.
4. The value of *x* is 29.5 because 16 + 13.5 is 29.5.

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

### MA.5.AR.2.4

Benchmark

MA.5.AR.2.4 Given a mathematical or real-world context, write an equation involving any of the four operations to determine the unknown whole number with the unknown in any position.

Example: The equation can be used to represent that sheets of paper are given to students from a pack of paper containing sheets with sheets left over.

Benchmark Clarifications:

*Clarification 1:* Instruction extends the development of algebraic thinking where the unknown letter is recognized as a variable.

*Clarification 2:* Problems include the unknown and different operations on either side of the equal sign.

Connecting Benchmarks/Horizontal Alignment

* MA.5.NSO.2.1, MA.5.NSO.2.2
* MA.5.AR.1.1

MA.5.AR.3.1Terms from the K-12 Glossary

* Equal Sign
* Equation
* Expression
* Whole Number

Vertical Alignment

|  |  |
| --- | --- |
| **Previous Benchmarks**   * MA.4.AR.2.2 | **Next Benchmarks**   * MA.6.AR.1.4 * MA.6.AR.2 |

Purpose and Instructional Strategies

The purpose of this benchmark is for students to write equations that determine unknown whole numbers from mathematical and real-world contexts. In Grade 4, students wrote equations from mathematical and real-world contexts to determine unknown whole numbers (represented by letter symbols) (MA.4.AR.2.2). The extension in Grade 5 is that factors are not limited to within 12 and equations may use parentheses, implying students may have to use the order of operations to solve. In Grade 6, students extend this work to include integers and positive fractions and decimals (MA.6.AR.2.2/2.3/2.4).

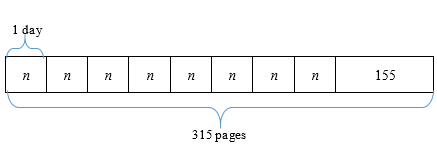
* Instruction focuses on helping students translate mathematical and real-world contexts to equations. Instructional emphasis should be placed on students’ comprehension of the contexts to then translate to equations more easily. An instructional strategy that helps students translate from context to symbolic equations is to first present contexts with some or all their numerical information omitted. In a mathematical context, this may look like showing a data display with some numerical information covered. In a real-world context, this may look like a word problem with quantities covered. This allows students to comprehend what the problem is trying to find and allows students to think deeper about what operations will be required to do so. It can also help students estimate reasonable solution ranges. Once students can predict an equation (or equations) to solve the problem, then the teacher can reveal all numerical information and allow students to solve *(MTR.5.1)*.
* In each context, students may provide many examples of equations that can be used to solve. During instruction, teachers can have students compare their equations and evaluate whether they can be used to solve *(MTR.4.1)*.
* During instruction, students can engage in discussions to justify how their equations match the mathematical and real-world contexts through checking solutions (*MTR.2.1*) Students may substitute their solution for their letter symbol and use the order of operations to check that it makes the equation true. Students are expected to explain how they can use algebraic equations to solve real-world solutions.

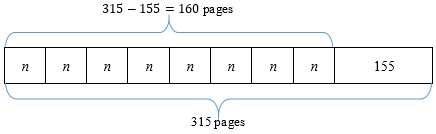
Common Misconceptions or Errors

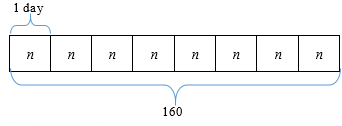
* When students have trouble comprehending contexts, they tend to just grab numbers from a given context and begin computing without justifying their arguments. Emphasis of instruction should be on the comprehension of problems through classroom discussion, sharing strategies, estimating reasonable solutions, and justifying equations and solutions (*MTR.2.1*).
* Students may overlook the fact that solving equations involves the application of inverse operations and the reverse order of operations. It is essential to revisit the concepts of the order of operations, the reverse order of operations, and the significance of inverse operations within the context of equations with students.

Strategies to Support Tiered Instruction

* Instruction focuses on the comprehension of problems through classroom discussion, sharing strategies, estimating reasonable solutions, and justifying equations and solutions.
* Instruction includes opportunities to connect real-world situations to write equations using any of the four operations to determine an unknown whole number with the unknown in any position. Students apply the order of operations to solve for the unknown. The teacher emphasizes the inverse relationships between addition and subtraction, and multiplication and division as applicable to help students solve for the unknown, while reinforcing conceptual understanding by having students use drawings, models and equations to solve real world problems.
  + For example, the teacher displays and reads the following problem aloud: “Renaldo read the same number of pages of his book each day for 8 days. He needs to read a total of 315 pages, and still needs to read 155 pages to meet his goal. How many pages did he read on each of the 8 days so far?” Students are provided manipulatives, such as counters or base-ten blocks, to model the problem or to use a drawing, such as a bar model, to solve and to write an equation. Through prompting and questioning, students explain their models, justify their solutions, and check their solution, repeating with multiple examples of real-world problems.

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* + For example, Elijah reads 25 pages of a novel per day for 7 days. The entire novel is 230 pages, how many pages does he have left to read? Students are provided manipulatives, such as counters or base-ten blocks, to model the problem or to use a drawing, such as a bar model, to solve and to write an equation. Through prompting and questioning, students explain their models, justify their solutions, and check their solution, repeating with multiple examples of real-world problems.

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Instructional Tasks

*Instructional Task 1 (MTR.7.1)*

To celebrate reaching their monthly reading goal, Dr. Ocasio’s class has a cookie party. Dr. Ocasio buys a box of 96 cookies. She plans to give the same number to each of the 21 students in her class. She wants 12 cookies remaining to bring home for her children. What is the greatest number of cookies each of Dr. Ocasio’s students can receive?

Part A. Write an equation that can be used to solve. Use a letter to represent the unknown number.

Part B. What is the greatest number of cookies each of Dr. Ocasio’s students can receive?

Part C. Prove that your answer is correct by showing how your equation is true.

*Instructional Task 2*

To celebrate the success of their science project, Mr. Anderson's class has a candy celebration. Mr. Anderson buys a bag of 120 candies. He plans to give the same number to each of the 18 students in his class. He wants 15 candies remaining to distribute later. What is the greatest number of candies each of Mr. Anderson's students can receive?

Part A: Write an equation that can be used to solve. Use a letter to represent the unknown number.

Part B: What is the greatest number of candies each of Mr. Anderson's students can receive?

Part C: Prove that your answer is correct by showing how your equation is true.

*Instructional Task 3*

Translate the written description to an equation. Then, find the value of *n.*

“The sum of 44 and 8 is equal to the product of 6 and an unknown number, *n*”

*Instructional Task 4*

Mario’s family goes to the movie theater. They buy two adult tickets for $13 each and three children’s tickets for $9 each. They also spend some money on popcorn. Altogether, they spent $57.

Part. A. Write an equation that can be used to find the cost of the popcorn. Use *p* to represent the unknown value.

Part B. What is the cost of the popcorn?

Part C. Check your answer using a strategy of your choice.

Instructional Items

*Instructional Item 1*

Which of the equations can be used to solve the problem below?

To celebrate reaching their monthly reading goal, Dr. Ocasio’s class has a cookie party. Dr. Ocasio buys a box of 96 cookies. She plans to give the same number to each of the 21 students in her class. She wants 12 remaining to bring home for her children. What is the greatest number of cookies each of Dr. Ocasio’s students can receive?



*Instructional Item 2*

Students at Maple Middle School are fundraising for a field trip, and they need to collect a total of $800. Currently, they have $250 from a bake sale. They plan to sell raffle tickets for $5 each to raise more funds. Write an equation to represent the situation, solve for the variable which represents the number of raffle tickets they need to sell to reach their goal, and explain the steps to find the value of

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

**MA.5.AR.3***Analyze patterns and relationships between inputs and outputs.*

### MA.5.AR.3.1

Benchmark

MA.5.AR.3.1 Given a numerical pattern, identify and write a rule that can describe the pattern as an expression.

Example: The given pattern can be described using the expression , where ; the expression , where or the expression , where .

Benchmark Clarifications:

*Clarification 1:* Rules are limited to one or two operations using whole numbers.

Connecting Benchmarks/Horizontal Alignment

* MA.5.AR.2.1, MA.5.AR.2.4

Terms from the K-12 Glossary

* Expression

Vertical Alignment

|  |  |
| --- | --- |
| **Previous Benchmarks**   * MA.4.AR.3.2 | **Next Benchmarks**  MA.6.AR.3.3 |

Purpose and Instructional Strategies

The purpose of this benchmark is for students to identify and write an expression that shows the rule for a given pattern. Students have been identifying and generating patterns since Grade 3. In Grade 5, the expectation builds on student knowledge of translating numerical expressions into written mathematical descriptions and extends to students writing a rule as an expression that may have 1 or 2 operations. In Grade 6, the focus is on patterns involving ratios (MA.6.AR.3.3).

* The rules for given patterns are limited to one or two operations using whole numbers.
* Vocabulary (e.g., coefficient, terms, variables) should be interwoven into the instruction of this benchmark. These terms are introduced in Grade 5, but not expected to be mastered until Grade 6.
* Students should understand that determining a rule for patterns helps them determine the value of future terms in the pattern *(MTR.2.1, MTR.5.1)*.
* Instruction includes showing students multiple ways to express multiplication. Grade 5 is the first time students may see multiplication expressed in these ways.
  + For example, students will need to build understanding of reasoning with multiplication expressions such as or
* During instruction, teachers can have students compare their rules and justify them using properties of operations.
  + For example, have students determine why the rule for the pattern in the benchmark example could be or *(MTR.5.1, MTR.6.1)*.
* Instruction of this benchmark should be paired with MA.5.AR.3.2. The combination of determining rules and completing tables is important for students to begin understanding ratios and functions in the middle grades *(MTR.5.1)*.
* Instruction includes recognizing patterns that arise from geometrical figures with different lengths and their perimeter or area.
  + For example, a pattern can arise from the following sequence of rectangles: 1 unit by 1 unit, 1 unit by 2 units, 1 unit by 3 units, 1 unit by 4 units. Students can describe the pattern of the perimeter or of the area.

Common Misconceptions or Errors

* Students may focus on a rule that works for a single input and its corresponding output. Remind students that the rule must work for all terms in a table.
  + For example, students may determine a rule based on the change in only the first two terms. During instruction, teachers should emphasize that a rule must work for the change in any two terms in a pattern.

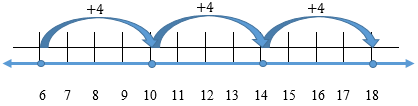
Strategies to Support Tiered Instruction

* Instruction includes opportunities to determine a rule given a numerical expression. After determining the rule, teachers provide guidance to support students as they work to describe the pattern as an expression. Special attention should be given to ensure that the rule is based on changes in all terms within the pattern (not just the first two terms).
  + For example, the teacher provides students with the first four terms of a pattern:

The teacher guides students to notice what pattern they see between the four terms (each number is five greater than the previous number). If students have difficulty, a number line or hundreds chart may be used to support finding the pattern. Students should identify that the rule is to add five. Based on this rule, the teacher guides students to represent the pattern as an expression (e.g., , where ) having students use the expression to check for accuracy with each of the terms in the pattern and identify the next two terms in the pattern ().

* + For example, the teacher provides students with the first four terms of a pattern.

The teacher guides students to notice what patterns they see between the four terms (each number is five greater than the previous number). A number line or hundreds chart is used to support finding the pattern. Students identify that the rule is to add four. Based on this rule, the teacher guides students to represent the pattern as an expression (e.g., , where ) having students use the expression to check for accuracy with each of the terms in the pattern and identify the next two terms in the pattern ().

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Instructional Tasks

*Instructional Task 1 (MTR.5.1)*

The first four terms of a pattern are below.

Part A. Write a mathematical description for a rule that matches these terms where

Part B. Write an expression that describes your rule.

Part C. Use your answer from Part B to determine the value of the 16th term.

*Instructional Task 2*

Write an expression to describe the rule for the pattern where

*Instructional Task 3*

Alexis is booking a vacation rental for $50 per night.

Part A. Write an expression to determine the total cost of renting the vacation home for *n* nights.

Part B. What would be the total cost for a 10-night stay?

*Instructional Task 4*

Jordan is reserving a conference room for $75 per hour.

Part A. Write an expression to calculate the total cost of renting the conference room for *h* hours.

Part B. What would be the total cost to rent the conference room for 9 hours each on three days?

Instructional Items

*Instructional Item 1*

Write an expression that can be a rule for the terms shown below where

*Instructional Item 2*

Use the pattern to answer the questions.

Part A. Identify and write the rule to describe the pattern where

Part B. Identify and write the rule to describe the pattern where

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

### MA.5.AR.3.2

Benchmark

MA.5.AR.3.2 Given a rule for a numerical pattern, use a two-column table to record the inputs and outputs.

Example: The expression , where represents any whole number, can be represented in a two-column table as shown below.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Input | 0 | 1 | 2 | 3 |
| Output | 6 | 8 | 10 | 12 |

Benchmark Clarifications:

*Clarification 1:* Instruction builds a foundation for proportional and linear relationships in later grades.

*Clarification 2:* Rules are limited to one or two operations using whole numbers.

Connecting Benchmarks/Horizontal Alignment

* MA.5.GR.4.2
* MA.5.DP.1.1

Terms from the K-12 Glossary

* Equation
* Whole number

Vertical Alignment

|  |  |
| --- | --- |
| **Previous Benchmarks**   * MA.4.AR.3.2 | **Next Benchmarks**   * MA.6.AR.3.3 |

Purpose and Instructional Strategies

The purpose of this benchmark is to relate patterns to a two-column table for students to record inputs and outputs. It is related to MA.5.AR.3.1 where students determine rules from given patterns. This is the first grade in which students record inputs and outputs two-column tables, and this work helps build the foundation for proportional relationships (MA.6.AR.3.3 and MA.7.AR.4) in middle school and functional relationships starting in Grade 8.

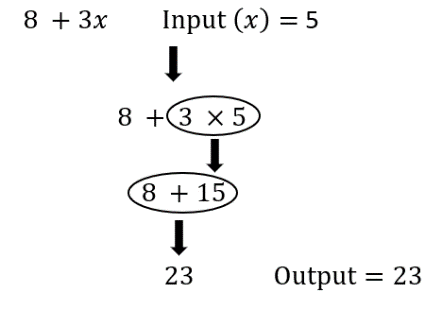
* Instruction of this benchmark should be paired with MA.5.AR.3.1. Organizing patterns into input and output tables lays the foundation for students to explore proportional and linear relationships in later grades *(MTR.5.1)*.
* During instruction, teachers can relate the idea of “inputs” and “outputs” on a two-column table to a machine. The input is the term number, and the output is the corresponding term’s value. Students are to find what the machine does to determine the output. Outputs can be generated from a given rule and set of inputs (*MTR.7.1*).
* Instruction should make connections between representing the information in a two-column table and as ordered pairs on a coordinate plane (MA.5.GR.4.2).
* Using tables to represent input/output tables prepares students to collect and represent data in line graphs (MA.5.DP.1.1).
* Instruction includes showing students multiple ways to express multiplication. Grade 5 is the first time students may see multiplication expressed in these ways.
  + For example, students will need to build understanding of reasoning with multiplication expressions such as or .

Common Misconceptions or Errors

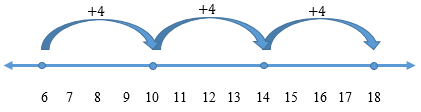
* Students may make computational errors when calculating the output for a given rule and input.
* When noting values in a two-column table, students may confuse input and output values.
  + For example, some students may incorrectly identify the equations in the output column as the actual outputs. It's important to highlight that the numeric outputs in the tables represent the solutions to the provided equations.
  + The substitution of numeric inputs into a rule helps to determine the corresponding outputs. An input/output table serves as a tool for organizing inputs and their respective outputs.

Strategies to Support Tiered Instruction

* Instruction includes opportunities to record each step when calculating the output for a given rule and input.
  + For example, for the rule students record the steps to calculate the output using an input of 5 and the order of operations.



* Instruction includes using highlighters when recording inputs and outputs in a two-column table. Students highlight the “inputs” label in the table and all corresponding inputs using one color. Then, students highlight the “outputs” label in the table and all corresponding outputs using a different color.

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Instructional Tasks

*Instructional Task 1 (MTR.5.1)*

The Math Machine makes two-column tables when the user tells it a rule. Jacob tells the Math Machine to create a table using the rule “” Unfortunately, the machine is malfunctioning and only some of the table is correct.

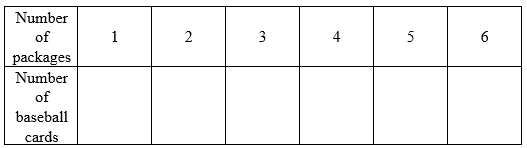
Part A: Identify which values are incorrect and complete the table correctly.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Input (*x*) | 0 | 1 | 2 | 3 |
| Output | 12 | 12 | 22 | 32 |

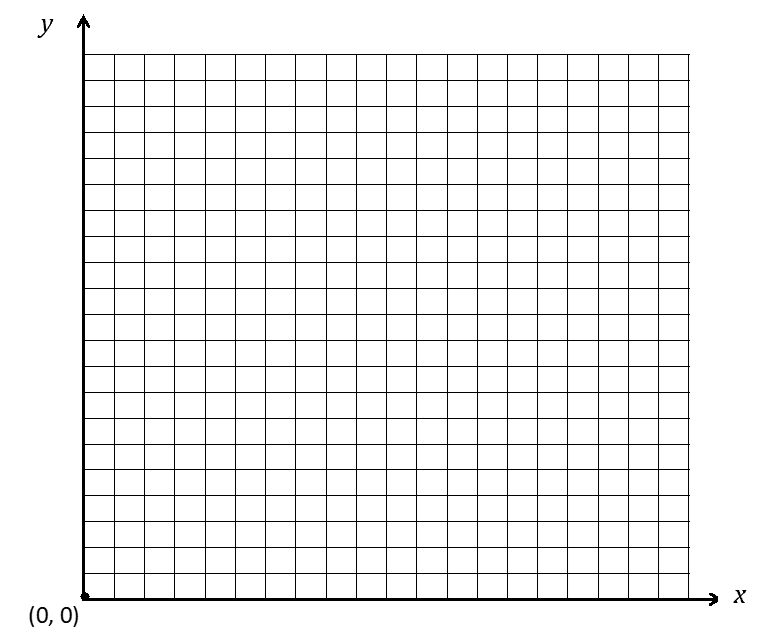
Part B: Extend your table to show the outputs for and .

*Instructional Task 2*

Vince collects baseball cards that come in packages of 3. Complete the table below to show how many baseball cards Vince would have for each number of packages he buys.



* + 1. Use the data in the table to form ordered pairs in which the input (*x*) is the number of packages, and the output (*y*) is the related number of baseball cards Vince has.
    2. Graph the ordered pairs on the coordinate grid below.



Instructional Items

*Instructional Item 1*

What is the missing value in the two-column table below?

Rule:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Input (*x*) | 0 | 1 | 2 | 3 |
| Output | ? | 37 | 34 | 31 |

*Instructional Item 2*

If each table in the U-shape arrangement can seat 4 people, what is the minimum number of tables required to accommodate a banquet for 40 people? 60 people? 180 people? Create a two-column table to record the inputs and outputs.

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

**Measurement**

**MA.4.M.1** *Measure the length of objects and solve problems involving measurement.*

### MA.4.M.1.1

Benchmark (Grade 4)

MA.4.M.1.1 Select and use appropriate tools to measure attributes of objects.

Benchmark Clarifications:

*Clarification 1:* Attributes include length, volume, weight, mass and temperature.

*Clarification 2:* Instruction includes digital measurements and scales that are not linear in appearance.

*Clarification 3:* When recording measurements, use fractions and decimals where appropriate.

Benchmark (Grade 5)

MA.5.M.1.1 Select and use appropriate tools to measure attributes of objects.

Benchmark Clarifications:

*Clarification 1:* Within the benchmark, the expectation is not to memorize the conversions.

*Clarification 2:* Conversions include length, time, volume and capacity represented as whole numbers, fractions and decimals.

Connecting Benchmarks/Horizontal Alignment.

* MA.4.FR.1.2/1.4
* MA.4.GR.1.2
* MA.4.DP.1.1
* MA.5.NSO.1.1
* MA.5.NSO.2.1/2.4/2.5
* MA.5.AR.1.2
* MA.5.AR.2.1
* MA.5.M.2.1
* MA.5.GR.1.1
* MA.5.GR.2.1
* MA.5.GR.3.3

Terms from the K-12 Glossary

Vertical Alignment

|  |  |
| --- | --- |
| **Previous Benchmarks**   * MA.3.M.1.1 * MA.4.M.1.2 | **Next Benchmarks**   * MA.6.AR.3.5 |

Purpose and Instructional Strategies

The purpose of this benchmark is to select and use tools to measure with precision. This concept builds on work to connect linear measurement to number lines. Students in the Grade 4 Accelerated course should be able to understand the relationship between units of measure through problem solving. This benchmark should be taught in conjunction with MA.4.M.1.2, where students acquire an understanding of conversion rules (e.g. multiply to change a larger unit to a smaller unit and divide to change a smaller unit to a larger unit.) This foundational concept is applicable to the conversion of any unit of measure. Additionally, in Grade 6 (MA.6.AR.3.5), students further enhance their understanding and establish connections between measurement conversion and the broader context of ratios and rates. During Grade 5, students utilize ratio reasoning to perform unit conversions. The application of ratio reasoning relies on procedures learned through the multiplication and division of fractions, skills that prove valuable in the context of measurement conversion.

Instruction allows students to convert measurements flexibly.

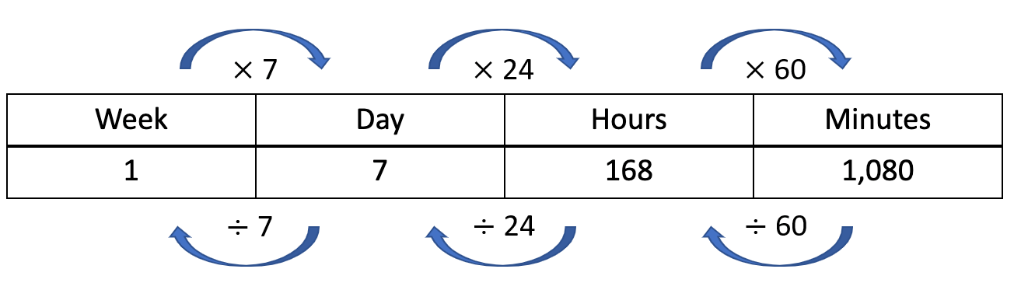
* + For example, when finding the number of inches in 2 yards, students may start with inches, feet or yards when calculating. Classroom discussion should compare those conversions to explore their similarities and differences *(MTR.2.1, MTR.4.1)*.
* For students to have a better understanding of the relationships between units, it is important for teachers to allow students to have practice with tools during instruction. This will show students how the number of units relates to the size of the unit.
  + For example, for students to discover converting inches to yards, teachers can have them use 12-inch rulers and yardsticks. This will allow students to see that three of the 12-inch rulers are equivalent to one yardstick (; ), so that students understand that there are 12 inches in 1 foot and 3 feet in 1 yard. Using this knowledge, students will be able to determine whether to multiply or divide when making conversions *(MTR.2.1)*.
* When moving into real-world problem solving, it is important to begin with problems that allow for renaming the units to represent the solution before using problems that require renaming to find the solution *(MTR.7.1)*.
* During instruction, teachers can integrate the relationship between metric units to support student understanding of the place value system. Students utilize their understanding of the patterns of multiplying and dividing with zeros when working with metric conversions. For students to build understanding of metric units, students can list units in order from greatest to least and practice conversion in relation to the place value position.
* For students to have a better understanding of measurement conversions and proportional relationships, allow students to have exposure to attributes of items being measured. Students should develop a foundation of the relative size of each unit and should have practice converting from a small unit to a larger unit and from a large unit to a smaller unit.

Common Misconceptions or Errors

* Students who struggle to identify benchmarks on number lines can also struggle to measure units of length, liquid volume, weight, mass and temperature. To assist students with this misconception, during instruction teachers should allow students to measure often and provide feedback. Students can also complete error and reasoning analysis activities to identify this common measurement misconception.
* Students confuse renaming units of measurement with the renaming that they do with whole numbers and place value.
  + For example, when subtracting 6 inches from 3 feet, they get 2 feet 4 inches because they think of subtracting 6 inches from 30 inches. Students need to pay attention to the unit of measurement which dictates the renaming (inches in this example) and the number to use (12 inches in a foot instead of 10 inches in a foot).
* Students confuse renaming units of measurements when converting mixed measures.
  + For example, when adding 3 feet 5 inches to 4 feet 9 inches, they get 7 feet 14 inches. Students may not understand that 14 inches can be converted into 1 foot 2 inches, for an overall answer of 8 feet 2 inches.
* Students may not know which operation to use when converting measurements. During instruction, teachers can use manipulatives and a real-world approach so that students make connections to units. For example, teachers can use containers that equal cups, pints, quarts, and gallons when giving explicit instruction on units of liquid capacity. Real- world application allows students to have a deeper understanding of the relative size of units that may be abstract.
* A common mistake that students may make when encountering word problems is when problems contain information in different units. Students should convert different units to one common unit before calculating.

Strategies to Support Tiered Instruction

* Instruction includes opportunities to measure often and provide feedback. Use error and reasoning analysis activities to address common measurement difficulties.
* Instruction includes providing students with a variety of objects. Ask students which tool they would use to measure each object. Discussions would include asking which attribute of the object is to be measured.
  + For example, objects could include a banana (where length or weight could be measured), water in a container (where temperature, volume or weight could be measured).
* Instruction includes deciding which operation to use when converting from smaller units to larger units (e.g., ounces to pounds) and when converting from larger units to smaller units (e.g., pounds to ounces). Instruction should also include estimating reasonable solutions.
  + For example, the teacher models a think aloud for which numbers to use based on the units of measurement and record the relationships on a chart.
    - How many minutes are in 1 week?
    - There are 60 minutes in 1 hour, 24 hours in 1 day and 7 days in 1 week. So, there are minutes in one week which is equivalent to 10,080 minutes.



* Instruction includes using a bar model or tape diagram to show the relationship between the units.
* Instruction includes opportunities to connect conversions of metric units to place value positions. Instruction includes utilizing place value mats when computing metric conversions to connect understanding of the relationship between the metric units. Students who struggle with multiplying and dividing across decimal place values should be provided with opportunities.
* Instruction includes opportunities to connect conversions of time to fractional parts such as halves and quarters. Instruction allows students to visualize parts of a clock in connection to real-world time application,
* Enrichment opportunity includes using a proportion to solve conversion items within the same measurement system.
  + Students can use a proportional ratio to create equivalent parts of a whole. For example, the teacher asks to convert 3 gallons into cups. There are 16 cups in 1-gallon therefore 1/16 equals 3/x. Students solve to find the variable x.
* Students will need to have an understanding that when changing from a larger unit to a smaller unit, you divide to solve, and when changing from a smaller unit to a larger unit, you multiply to solve.

Instructional Tasks

*Instructional Task 1 (MTR.7.1)*

Use a thermometer to measure the temperature to the nearest 0.1 degree Fahrenheit at 8:30 a.m., 11:00 a.m. and 1:30 p.m. every day for one week. Record each temperature in a table.

*Instructional Task 2 (MTR.6.1, MTR.7.1)*

Zevah is helping her mom plan her sister’s surprise birthday party.

Part A. The recipe to make one bowl of punch is shown below. How many cups of punch will they be able to serve at the party if they only make one bowl of punch and there is no punch leftover in the bowl?

| **Liquid** | **Fluid Ounces** |
| --- | --- |
| Pineapple Juice | 32 oz |
| Fruit Punch | 64 oz |
| Ginger Ale | 76 oz |

Part B. At the party, Zevah wants each balloon to have a string that is 250 centimeters long. The string she wants to buy comes in rolls of 30 meters. How many rolls of string does Zevah need to buy if she plans to have 36 balloons at the party?

*Enrichment Task 1 (MTR.3.1)*

How do you convert 108 inches to yards?

*Enrichment Task 2 (MTR.5.1)*

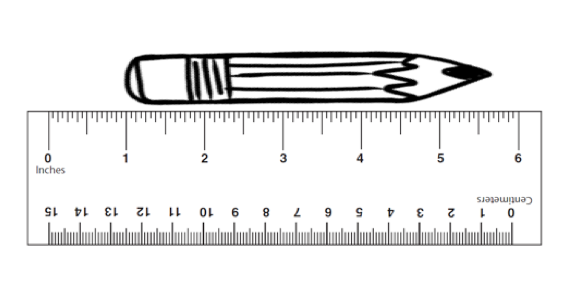
Select all the measurements less than 435 inches.

1. 37 feet
2. 36 feet 2 inches
3. 12 yards 3 inches
4. 12 feet 3 inches
5. 30 feet

Instructional Items

*Instructional Item 1*

A pencil is shown. Using the ruler provided, what is the length of the pencil to the nearest inch?



*Instructional Item 2*

Michael is measuring fabric for the costumes of a school play. He needs 11.5 meters of fabric. He has 280 centimeters of fabric. How many more centimeters of fabric does he need?

*Instructional Item 3*

A recipe requires 24 ounces of milk. Edwin has only a cup measuring cup. How many measuring cups of milk will Edwin need?

1. 6
2. 12
3. 18
4. 24

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

### MA.4.M.1.2

Benchmark

MA.4.M.1.2 Convert within a single system of measurement using the units: yards, feet, inches; kilometers, meters, centimeters, millimeters; pounds, ounces; kilograms, grams; gallons, quarts, pints, cups; liter, milliliter; and hours, minutes, seconds.

*Example:* If a ribbon is 11 yards 2 feet in length, how long is the ribbon in feet?

Example: A gallon contains 16 cups. How many cups are in gallons?

Benchmark Clarifications:

*Clarification 1:* Instruction includes the understanding of how to convert from smaller to larger units or from larger to smaller units.

*Clarification 2:* Within the benchmark, the expectation is not to convert from grams to kilograms, meters to kilometers or milliliters to liters.

*Clarification 3:* Problems involving fractions are limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12, 16 and 100.

Connecting Benchmarks/Horizontal Alignment

* MA.4.M.1.1

Terms from the K-12 Glossary

Vertical Alignment

|  |  |
| --- | --- |
| **Previous Benchmarks**   * MA.3.M.1.1 | **Next Benchmarks**  MA.5.M.1.1 |

Purpose and Instructional Strategies

The purpose of this benchmark is for students to see the relationships between the units they use for measurement. Students should begin to generalize that the smaller the unit is, the more precise measurement they will get, but will also need more of the unit to measure *(MTR.5.1)*. Work in this benchmark builds from Grade 3 foundations of using customary measurements (MA.3.M.1.1).

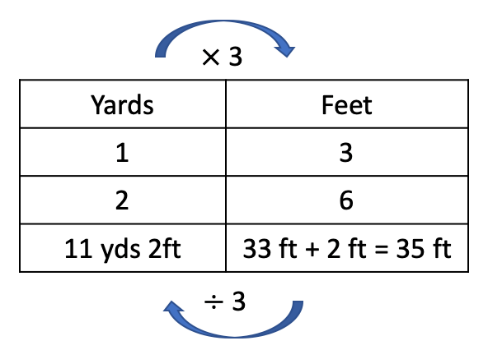
* For instruction, students need to use measuring devices in class to develop a sense of the attributes being measured to have a better understanding of the relationships between units.
* The number of units relates to the size of the unit. Students need to develop an understanding that there are 12 inches in 1 foot and 3 feet in 1 yard. Allow students to use rulers or a yardstick to discover these relationships among units of measurements. Using 12-inch rulers and yardsticks, students will see that three of the 12-inch rulers are the same length as a yardstick, so 3 feet is equivalent to one yard. A similar strategy can be used with rulers marked with centimeters and a meter stick to discover the relationships between centimeters and meters.
* To help students to visualize the size of units, they should be given multiple opportunities to measure the same object with different measuring tools.
  + For example, have the students measure the length of a room with one-inch tiles, with one-foot rulers, and with yardsticks. Students should notice that it takes fewer yard sticks to measure the room than rulers or tiles and explain their reasoning.
* During instruction, have students record measurement relationships in a two-column table or t-chart.
* Students are not expected to memorize conversions. Students should be provided conversion tools (e.g., charts) during instruction.

Common Misconceptions or Errors

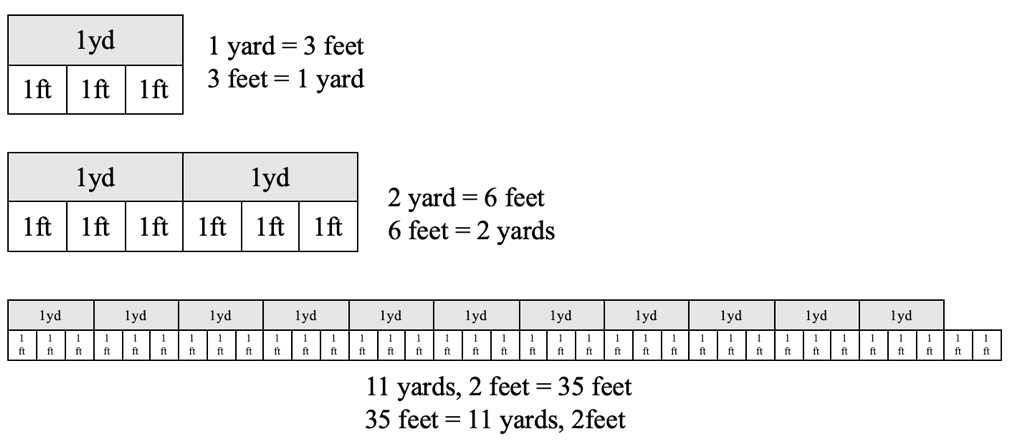
* Students can assume that converting from smaller units to larger units (e.g., ounces to pounds), that multiplication is used, and when converting from larger units to smaller units (e.g., pounds to ounces), that division is used. To assist students with this misconception, expect them to estimate reasonable solutions. Students will begin to acquire an understanding of conversion rules (e.g. multiply to change a larger unit to a smaller unit and divide to change a smaller unit to a larger unit), an extension of the work started in the previous benchmark MA.4.M.1.1.

Strategies to Support Tiered Instruction

* Instruction includes demonstrating which operation to use when converting from smaller units to larger units (e.g., ounces to pounds) and when converting from larger units to smaller units (e.g., pounds to ounces). Instruction also includes estimating reasonable solutions. The teacher models a think aloud and record the relationships on a two-column chart.
  + For example, “If a ribbon is 11 yards 2 feet in length, how long is the ribbon in feet?”
    - “I know that for every 1 yard, there are 3 feet. So, I can multiply the number of yards by 3 to convert to feet. I also know that for every 3 feet, there is one yard. I can divide the number of feet by 3 to convert to yards. Therefore 2 yards convert to 6 feet and 6 feet converts to 2 yards. To find out how many feet there are in 11 yards, 2 feet, I have to multiply the number of yards by 3. . Next, I have to add the extra 2 feet. So, 11 yards, 2 feet converts to 35 feet.”



* Instruction includes using a bar model or tape diagram to show the relationship between the units. The teacher models a think aloud.
  + For example, “If a ribbon is 11 yards 2 feet in length, how long is the ribbon in feet?”
    - “I know that for every 1 yard, there are 3 feet. So, I can multiply the number of yards by 3 to convert to feet. I also know that for every 3 feet, there is one yard. I can divide the number of feet by 3 to convert to yards. Therefore 2 yards convert to 6 feet and 6 feet converts to 2 yards. To find out how many feet there are in 11 yards, 2 feet, I have to multiply the number of yards by 3. . Next, I have to add the extra 2 feet. So, 11 yards, 2 feet converts to 35 feet.”



Instructional Tasks

*Instructional Task 1 (MTR.3.1)*

Calculate how many minutes there are in 1 week.

*Enrichment Task 1 (MTR.6.1)*

Every 20 seconds, a car passes Jenny’s house. Will it take more than an hour for 150 cars to pass by Jenny’s house?

*Enrichment Task 2 (MTR.7.1)*

Melanie stretches for 35 seconds and then sprints for 55 seconds. She repeats her workouts 9 times. If she does not take any breaks, how many minutes does she exercise for?

Instructional Items

*Instructional Item 1*

There are 3 paperclip chains. Chain A is 50 inches long, Chain B is feet long. Chain C is 1 yard long. Order the chains from the longest length to the shortest length.

1. Chain A, Chain B, Chain C
2. Chain B, Chain C, Chain A
3. Chain C, Chain B, Chain A
4. Chain B, Chain A, Chain C

*Instructional Item 2*

Darren grew a plant from seed. After 3 weeks, the plant grew 8 centimeters. How many millimeters did the plant grow?

1. 0.8
2. 8
3. 80
4. 800

*Enrichment Item 1 (MTR.5.1)*

How do you convert 3 1/3 yards to inches? Circle the correct answer for each step.

Step 1: There are \_\_\_\_\_ inches in 1 yard

1. 36
2. 12
3. 3

Step 2: I need to \_\_\_\_\_\_\_\_\_\_\_

1. Multiply
2. Divide

Step 3: 3 yards= \_\_\_\_\_\_\_ inches

1. 4
2. 15
3. 120

*Enrichment Item 2 (MTR.7.1)*

What is the perimeter of a pool with a length of 60 feet and a width of 25 yards?

1. 150 feet
2. 150 yards
3. 270 feet
4. 270 yards

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

**MA.4.M.2** *Solve problems involving time and money.*

### MA.4.M.2.1

Benchmark

MA.4.M.2.1 Solve two-step real-world problems involving distances and intervals of time using any combination of the four operations.

Benchmark Clarifications:

*Clarification 1:* Problems involving fractions will include addition and subtraction with like denominators and multiplication of a fraction by a whole number or a whole number by a fraction.

*Clarification 2:* Problems involving fractions are limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12, 16 and 100.

*Clarification 3:* Within the benchmark, the expectation is not to use decimals.

Connecting Benchmarks/Horizontal Alignment

* MA.4.M.1.2

Terms from the K-12 Glossary

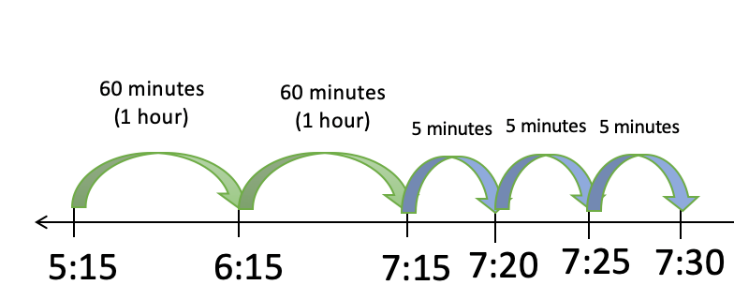
Vertical Alignment

|  |  |
| --- | --- |
| **Previous Benchmarks**   * MA.3.M.2.2 | **Next Benchmarks** |

Purpose and Instructional Strategies

The purpose of this benchmark is to connect concepts of unit conversions to time and distance and solve problems with these conversions. In Grade 3, students solved one- and two-step elapsed time problems without converting units of time or crossing from a.m. to p.m. or p.m. to a.m. (MA.3.M.2.2).

* For distance problems, students may need to understand multiplicative comparison (e.g., 20 is twice as many as 10).
* For instruction, an open number line is a strategy students can use to solve elapsed time problems.



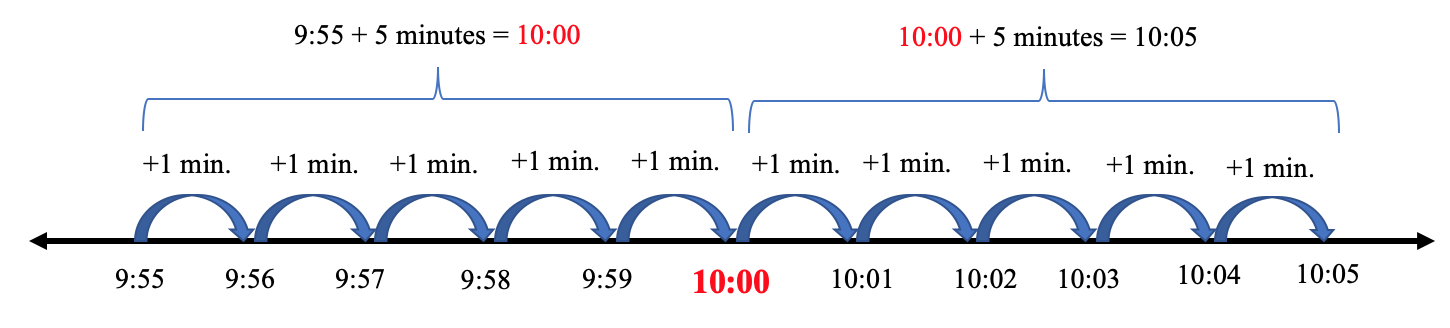
* Students need to spend time solving problems crossing between a.m. and p.m., and vice-versa.
* Students should also have a firm understanding of the terms quarter hour (15 minutes) and half hour (30 minutes).

Common Misconceptions or Errors

* Students can get confused when time crosses the hour because it does not follow the base-ten pattern where they are familiar. For example, students can misinterpret that the elapsed time between 9:55 a.m. and 10:05 a.m. and state that the elapsed time is 50 minutes because they have found the difference from 55 to 105. The use of number lines and clocks side-by-side help students build understanding about how elapsed time is calculated.

Strategies to Support Tiered Instruction

* Instruction includes the use of number lines and clocks side-by-side to help students build understanding about how elapsed time is calculated.
* Instruction includes using a number line and counting by ones to demonstrate what happens when time crosses the hour because it does not follow the familiar base ten pattern.
  + For example, use a number line to find the elapsed time between 9:55 a.m. and 10:05 a.m. and explain what happens when time crosses the hour at 10:00 a.m.



* Instruction includes demonstrating what happens when time crosses the hour because it does not follow the familiar base ten pattern.
* For example, instruction may include using a geared manipulative (Judy) clock to find the elapsed time between 9:55 a.m. and 10:05 a.m. Students move the minute of the hand one minute at a time from 9:55 to 10:00. After each minute, the teacher asks the students to record what time it is. The teacher has students pay special attention to what happens when the minute hand moves from 9:59 to the next minute.
* Instruction includes using the 3-Read Protocol to break down the problem. First read the problem to understand the context, possibly without the numbers. Then, read to understand the mathematics. Finally, read to create a plan for solving the problem.

Instructional Tasks

*Instructional Task 1 (MTR.7.1)*

Steve drove 2,465 miles away to college. On Parents’ Weekend, his parents drove the distance round trip from home, with an additional 385 miles traveled to visit his sister on their return trip. How many total miles did his parents drive on Parents’ Weekend?

*Instructional Task 2*

To prepare for track club, Miranda runs after school. Each day she trains, she completes three rounds of running for 4 minutes and 30 seconds. Between each round, she rests for 1 minute.

Part A. What is the total number of minutes Miranda runs each day she trains?

Part B. If she trained 4 days last week, what was the total number of minutes she trained for the week?

*Enrichment Task 1 (MTR.7.1)*

A television station shows commercials for 13 1/3 minutes each hour. How wany 45-second commercials can it show?

*Enrichment Task 2 (MTR.3.1)*

Hannah is training for a marathon. She is running laps around a football field. If the distance around the field is 600 yards, how many complete laps would she need to do to run at least 4 miles?

Instructional Items

*Instructional Item 1*

After lunch, Billy walked the dog for 17 minutes and then immediately after, did his chores for 58 minutes. If he finished his chores at 12:15 p.m., what time did he start walking the dog?

1. 1:30 p.m.
2. 1:13 p.m.
3. 11:17 a.m.
4. 11:00 a.m.

*Enrichment Item 1*

Which is the correct comparison?

1. 4.25 hours= 260 minutes
2. 260 minutes > 4.25 hours
3. 4.25 hours > 260 minutes
4. 260 minutes < 4.25 hours

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

### MA.4.M.2.2

Benchmark

MA.4.M.2.2 Solve one- and two-step addition and subtraction real-world problems involving money using decimal notation.

*Example:* An item costs $1.84. If you give the cashier $2.00, how much change should you receive? What coins could be used to give the change?

*Example:* At the grocery store you spend $14.56. If you do not want any pennies in change, how much money could you give the cashier?

Connecting Benchmarks/Horizontal Alignment

* MA.4.NSO.2.7

Terms from the K-12 Glossary

Vertical Alignment

|  |  |
| --- | --- |
| **Previous Benchmarks**   * MA.2.M.2.2 | **Next Benchmarks**  MA.5.M.2.1 |

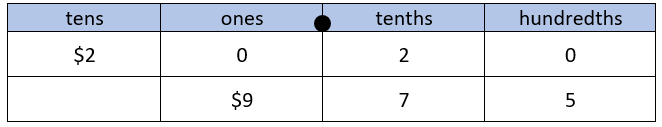
Purpose and Instructional Strategies

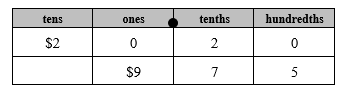
The purpose of this benchmark is to connect money concepts to adding and subtracting decimals. This benchmark can be taught in tandem with the addition and subtraction of decimals to the hundredths (MA.4.NSO.2.7). Students solve problems within a real-world context using money *(MTR.7.1)*. Money contexts are an important concept and build the foundation in later grades where students are expected to fluently use decimal operations to solve equations with variables (MA.6.NSO.2.3).

* For instruction, students should have opportunities using multiplication to count collections of coins (e.g., How much money is 50 nickels?).
* Instruction allows students to make computations flexibly using various strategies *(MTR.2.1)*
* When students solve problems, invite flexible strategies that students learned with whole number addition and subtraction. For example, when finding the change for $2.00 on an item that costs $1.84, students may count up $0.16 instead of subtracting $2.00 - $1.84.
* Students need to understand how different coins and bills relate to each other.

Common Misconceptions or Errors

* Instruction includes connecting place value to addition and subtraction of whole numbers, utilizing place value charts so that students can see where to line up values for the computation.
  + For example, $20.20 – $9.75 is going to require some regrouping. By placing the problem in a place value chart, students can line up the decimal and subtract like place values.



****

* Instruction includes relating decimal place values. Working with base ten blocks, students can build decimals and their equivalents.
  + For example, building 0.2 “two tenths” and 0.20 “twenty hundredths” with base ten blocks. Students will notice that the numbers have the same value.

|  |  |
| --- | --- |
| **2 tens rods** | **Twent unit cubes.** |
| 0.2 | 0.20 |

* Instruction includes using the 3-Read Protocol to break down the problem. First read the problem to understand the context, possibly without the numbers. Then, read to understand the mathematics. Finally, read to create a plan for solving the problem.
  + The teacher encourages and guides students to think backwards when completing word problems. Identify what the problem is asking and create action steps to solve the problem. For example, why do you subtract when a question asks you to find how much change will be given back after a purchase? Students should have an understanding that change back is the difference between the amount of money given and the cost of an item.
  + The teacher encourages students to utilize peer discourse to identify important information when solving word problems. Have students read word problems to a partner and explain to each other what steps are needed to solve the problem (*MTR.4.1*).

Instructional Tasks

*Instructional Task 1*

Jordan was saving his money to buy a remote control motorcycle. He saved $45.00 from his allowance and received two checks worth $10.00 each for his birthday. Jordan also has a half dollar coin collection with 30 coins in it. If the motorcycle costs $73.00, does Jordan have enough money to buy the motorcycle?

*Enrichment Task 1 (MTR.5.1)*

Remi has $4.50. She wants to buy two goldfish. The goldfish each cost $2.30. How much more money would she need to buy two goldfish?

*Enrichment Task 2 (MTR.7.1)*

A table with a price list
Soup $3.42, Salad $4.78, Fruit $1.64, Sandwich $4.62 and Drink $1.73

Ana and Devin buy 2 cups of soup, 1 salad, 3 pieces of fruit, 4 sandwiches, and 2 drinks. They give the cashier $40. What bills and coins could they receive in change? Draw a model to solve.

Instructional Items

*Instructional Item 1*

Maria went to the comic bookstore and bought a comic book for $5.34 and a comic book for $9.55. If she paid with a $20 bill, how much change would she get back?

*Instructional Item 2*

Boris has $42 to buy 3 games that each cost $12.56. How much change should he receive?

1. $3.32
2. $3.42
3. $4.32
4. $4.42

*Enrichment Item 1 (MTR.3.1)*

Nancy buys a new skateboard for $65.97 and a helmet for $37.65. How much does she spend?

1. $93.62
2. $103.62
3. $93.52

*Enrichment Item 2 (MTR.7.1)*

Claire buys 3 copies of the same video game to give to friends. Each video game costs $15.75. How much does Claire spend?

1. $18.75
2. $12.75
3. $47.25
4. $45.75

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

***MA.5.M.1*** *Convert measurement units to solve multi-step problems.*

### MA.5.M.1.1

### See Benchmark MA.4.M.1.1 (insert hyperlink here)

**MA.5.M.2***Solve problems involving money.*

### MA.5.M.2.1

Benchmark

MA.5.M.2.1 Solve multi-step real-world problems involving money using decimal notation.

*Example:* Don is at the store and wants to buy soda. Which option would be cheaper: buying one 24-ounce can of soda for $1.39 or buying two 12-ounce cans of soda for 69¢ each?

Connecting Benchmarks/Horizontal Alignment

* MA.5.NSO.1.1, MA.5.NSO.1.2, MA.5.NSO.1.3
* MA.5.NSO.2.3, MA.5.NSO.2.4, MA.5.NSO.2.5
* MA.5.AR.2.1, MA.5.AR.2.4
* MA.5.M.1.1

Terms from the K-12 Glossary

Vertical Alignment

|  |  |
| --- | --- |
| **Previous Benchmarks**   * MA.4.M.2.2 | **Next Benchmarks**   * MA.6.NSO.2.3 |

Purpose and Instructional Strategies

The purpose of this benchmark is for students to apply understanding of multi-step real-world problems, measurement conversions, and decimal operations to solve problems involving money *(MTR.7.1)*. This benchmark connects to previous work in Grade 4 where students added and subtracted money in real world situations (MA.4.M.2.2). Money contexts continue to be important throughout the later grades. In Grade 6, students are expected to fluently use decimal operations to solve equations with variables (MA.6.NSO.2.3).

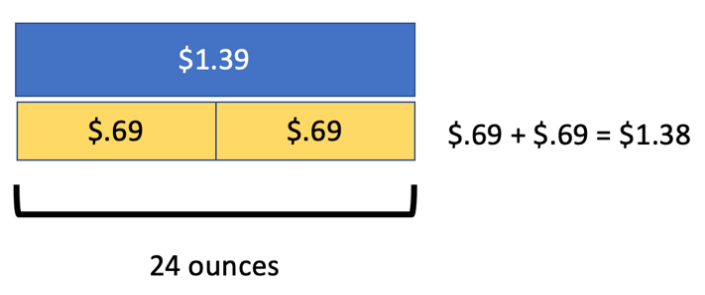
* During instruction, teachers should provide strategies for helping students build mathematical literacy, increasing comprehension and problem-solving skills so that students can identify important information in a word problem before completing numerical calculations. Students should be encouraged to estimate a solution and model a problem using manipulatives, pictures and/or equations before computing *(MTR.2.1)*.
* In this benchmark, money is expressed as decimal numbers, so decimal operations can be used to solve problems involving money. Instruction allows students to make computations flexibly using various strategies *(MTR.2.1)*

Common Misconceptions or Errors

* Students can misinterpret multi-step word problems and only complete one of the steps. Encourage students to estimate reasonable solutions and justify models to solve before computing.
* Students may get confused when reading word problems. Teachers should encourage students to draw and label diagrams with monetary value as a visual aid.

Strategies to Support Tiered Instruction

* Instruction includes encouraging students to estimate reasonable solutions and justify models before performing computations of a multi-step word problem.
  + Teachers have the opportunity to observe student behaviors as evidence that students are checking for reasonableness while exhibiting proficiency with mathematical thinking and reasoning standards. When assessing for reasonableness students should actively estimate to discover and explore possible solutions to problems. Students should use benchmark quantities to determine if a solution makes sense. Students should also check calculations during and after calculations when solving a problem and they verify possible solutions by explaining the methodology used. When assessing reasonableness, students can analyze results of a problem based on the given context before computation (*MTR.6.1*).
* Instruction includes using visual models, such as bar models or tape diagrams, to help to visualize the problem.
  + For example, which is a better deal, buying one 24oz. can for $1.39 or two 12 oz. cans for $0.69 each?



* Instruction includes visualizing word problems. The Three-Reads Protocol is a strategy that can be used to help students conceptualize what the question is asking. Students draw pictures or models to represent what is happening in the word problem. These pictures and models can be used to help students write equations for the problem they are solving.
* Instruction includes breaking down word problems into smaller parts. Students use a highlighter to emphasize the important information in the word problem and paraphrase the word problem so the teacher can determine if the student understands what the question is asking.
  + The teacher encourages and guides students to think backwards when completing word problems. Identify what the problem is asking and create action steps to solve the problem. For example, why do you subtract when a question asks you to find how much change will be given back after a purchase? Students should have an understanding that change back is the difference between the amount of money given and the cost of an item.
  + The teacher encourages students to utilize peer discourse to identify important information when solving word problems. Have students read word problems to a partner and explain to each other what steps are needed to solve the problem (*MTR.4.1*).

Instructional Tasks

*Instructional Task 1 (MTR.7.1)*

Jordan was saving his money to buy a remote control motorcycle. He saved $37.81 from his allowance and received two checks worth $10.00 each for his birthday. Jordan also has a half dollar coin collection with 30 coins in it. If the motorcycle costs $72.29, does Jordan have enough money to buy the motorcycle?

*Instructional Task 2*

A restaurant owner is buying ice. She needs 60 pounds. Should she buy the 20-pound bags of ice for $4.78 or the 10-pound bags of ice for $2.75 each?

*Instructional Task 3*

Maria is a college student managing her monthly budget. This month, she has a part-time job that pays her $12.65 per hour, and she worked 20 hours. Additionally, she received $50 as a birthday gift from her grandparents. On the spending side, she had to pay $29.95 for a textbook, $94.10 for groceries, and $15.50 for transportation.

Can you help Maria calculate her total earnings for the month and determine if she has enough money to cover her expenses? If there is any money left, how much does she have saved or, if she is short, how much does she need to cover her expenses?

*Instructional Task 4*

Lily and Jake decided to start a small business selling handmade candles. They invested $350.25 in purchasing materials and equipment. After creating and selling the candles, they earned a total revenue of $625.80. However, they incurred additional costs for packaging and transportation, amounting to $89.50.

Determine the amount of money made by Lily and Jake. Write your answer in decimal notation.

Instructional Items

*Instructional Item 1*

Pecans and almonds each cost $6.80 per pound. Kendall buys 1.5 pounds of pecans and 2.5 pounds of almonds. What is the total cost of Kendall’s purchase?

*Instructional Item 2*

A table below shows the costs of items at a candy store.

|  |  |
| --- | --- |
| **Item** | **Cost** |
| Chocolate bar | $2.99 each |
| Candy rope | $0.45 per ounce |
| Peanut butter cups | $1.50 each |
| Bubble gum | $0.29 per ounce |

Wayne has $10 to spend. Select all the purchases that Wayne has enough money to make.

1. 3 chocolate bars
2. 25 ounces of candy rope
3. 2 chocolate bars and 3 peanut butter cups
4. 3 peanut butter cups and 5 ounces of bubble gum
5. 24 ounces of bubble gum and 2 ounces of candy rope

*Instructional Item 3*

Ronald goes to dinner with some friends and pays the bill. Each person orders a meal that costs $14.95. Ronald gave the server a $11.96 tip. The total bill, including the tip, was $71.76. How many friends did Ronald pay for?

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

## Geometric Reasoning

**MA.5.GR.1***Classify two-dimensional figures and three-dimensional figures based on defining attributes.*

### MA.5.GR.1.1

Benchmark

MA.5.GR.1.1 Classify triangles or quadrilaterals into different categories based on shared defining attributes. Explain why a triangle or quadrilateral would or would not belong to a category.

Benchmark Clarifications:

*Clarification 1:* Triangles include scalene, isosceles, equilateral, acute, obtuse and right; quadrilaterals include parallelograms, rhombi, rectangles, squares and trapezoids.

Connecting Benchmarks/Horizontal Alignment

* There are no direct connections outside of this standard; however, teachers are encouraged to find possible indirect connections.

Terms from the K-12 Glossary

* Acute Triangle
* Equilateral Triangle
* Isosceles Triangle
* Obtuse Triangle
* Parallelograms
* Quadrilateral
* Rectangle
* Rhombus
* Right Triangle
* Scalene Triangle
* Square
* Trapezoid
* Triangle

Vertical Alignment

|  |  |
| --- | --- |
| **Previous Benchmarks**   * MA.4.GR.1.1 | **Next Benchmarks**  MA.912.GR.3.2 |

Purpose and Instructional Strategies

The purpose of this benchmark is for students to understand that shapes can be classified by their attributes and these attributes may place them in multiple categories. Attributes of figures refer to its traits (such as angle classifications, numbers of sides, side lengths). Students have previous experience describing attributes of figures since Kindergarten. In Grade 3, students identified and drew quadrilaterals based on their attributes (MA.3.GR.1.2). In Grade 4, students explored angle classifications and measures in two-dimensional figures (MA.4.GR.1.1). This past work built the understanding required for students to classify triangles and quadrilaterals in Grade 5. Classification of geometric figures will return in high school geometry (MA.912.GR.3.2) using another Grade 5 concept, the coordinate plane.

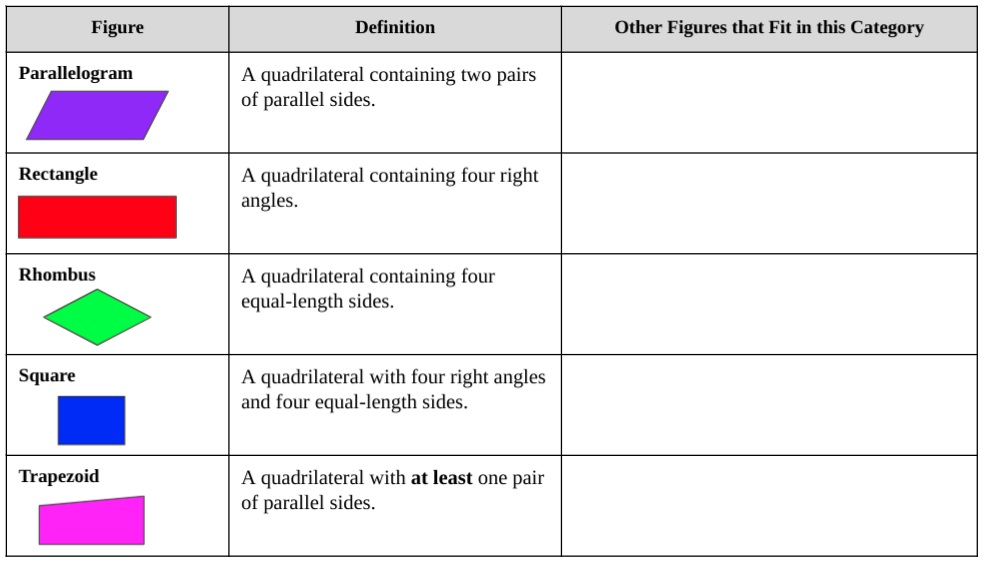
* The work in Grade 5 will help students to understand that triangles can be defined by two different attributes that students can measure: the length of their sides (3 congruent sides, 2 congruent sides, or 0 congruent sides) and the size of their angle measures (3 acute angles, 2 acute angles and a right angle, or 2 acute angles and an obtuse angle).
* During instruction, it is important for students to have practice with classifying figures in multiple ways so they can better understand the relationship between attributes of the geometric figures. In addition, students should practice this concept by using graphic organizers such as flow charts, T-charts and Venn diagrams *(MTR.2.1)*.
* This benchmark requires a strong understanding and use of geometry vocabulary. Allow students to use math discourse throughout instruction to compare the attributes of geometric figures.
  + For example, pose questions such as, “Why is a square always a rhombus?” and “Why is a rhombus not always a square?” Lesson activities may require students to justify their thinking when making mathematical arguments about geometric figures *(MTR.4.1)*.

Common Misconceptions or Errors

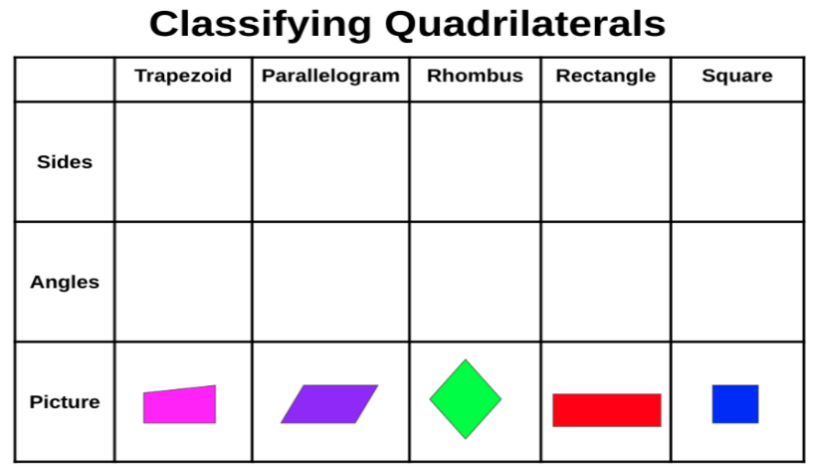
* Students may think that when describing and classifying geometric shapes and placing them in subcategories, the last subcategory is the only classification that can be used.
* Students may think that a geometric figure can only be classified in one way.
  + For example, a square (a shape with 4 congruent sides and 4 congruent angles) can also be a parallelogram because it contains 2 pairs of sides that are congruent and parallel.

Strategies to Support Tiered Instruction

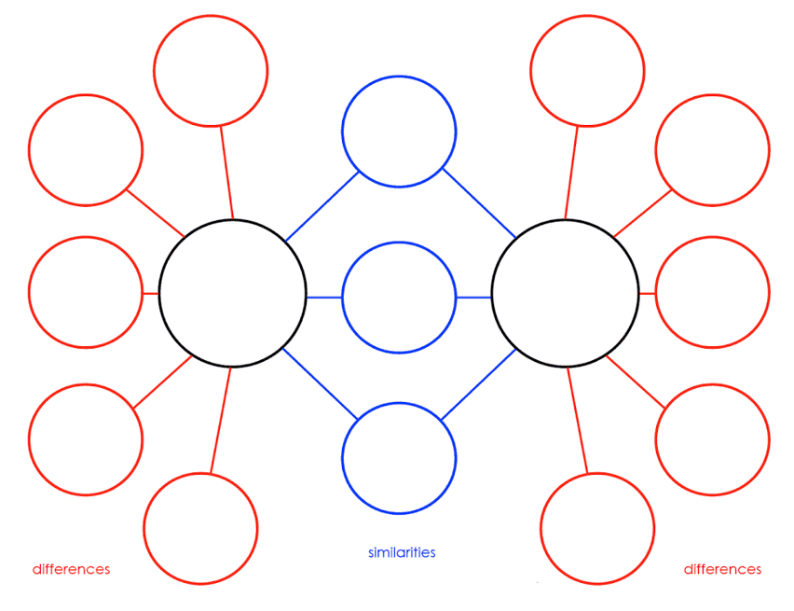
* Instruction includes providing a graphic organizer and having students place triangles and/or quadrilaterals into all the subcategories they belong to. Students then identify all the ways the figure could be classified.
  + For example, students are provided with a graphic organizer like the one shown below to help them classify figures into subcategories. The name of the figure, an example, and the definition are provided. Students then identify which other categories the figure would also fit. For example, a parallelogram is a quadrilateral containing two pairs of parallel sides. A rectangle, rhombus, and square all also have two pairs of parallel sides so they would also fit in this subcategory. The teacher refers to the glossary, included with the standards, for several examples to provide students.

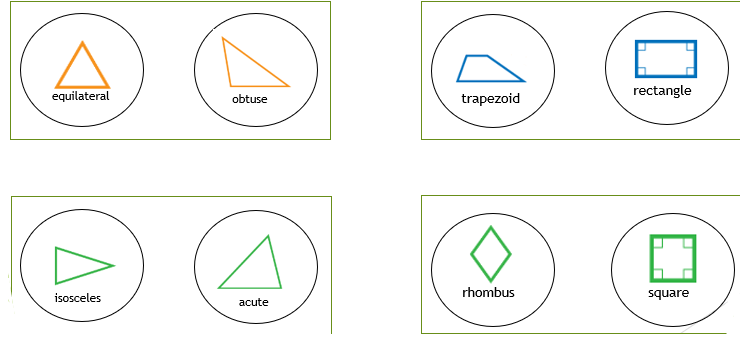


* Instruction includes providing a graphic organizer and having students use sticky notes with specific attributes on them to help them classify figures.
  + For example, students are provided with a graphic organizer like the one shown below with an example of the figure filled in for them to refer to and yellow sticky notes that have “4 equal sides” written on them. Students determine which figures contain this attribute and place the sticky note under those figures (square and rhombus). The teacher then provides green sticky notes with “two pairs of parallel sides” written on them. Students place the sticky note under each figure that has that attribute (parallelogram, rhombus, rectangle, and square). Students would continue to add different color sticky notes with attributes that say, “One pair of parallel sides” and “four right angles”. Students are able to see that some figures have several sticky notes and which figures have the same sticky notes. Students will then name all the ways a figure can be classified based on the attributes they have.



* Instruction may include providing a variety of graphic organizers that allow students to build physical and cognitive connections.
  + For example, have students create or provide them with a template to complete a double-bubble graphic organizer; then allow them to compare and contrast assorted examples.





Instructional Tasks

*Instructional Task 1*

When can a rectangle be a rhombus? Can a rhombus be a rectangle? Explain using examples or drawings.

*Instructional Task 2*

A pool is shaped like a rhombus with a side length of 6 meters. What is the perimeter of the pool?

*Enrichment Task 1*

Part A. Have students draw and label five different quadrilaterals on a small piece of paper: A parallelogram, a rectangle, a rhombus, a square, and a trapezoid.

Part B. Have students create a quadrilateral organizational tree. Ask students advancing questions such as “Is a trapezoid a parallelogram?” or “Is a rectangle a parallelogram or a trapezoid?” to help students identify the relationship between quadrilaterals and their tiers.

Instructional Items

*Instructional Item 1*

Choose all the shapes that can **always** be classified as parallelograms.

* 1. Trapezoid
  2. Rectangle
  3. Rhombus
  4. Square
  5. Equilateral Triangle

*Enrichment Item 1*

Which statement is always true?

a. Trapezoids are parallelograms.

b. A rhombus is a square.

c. A rhombus has sides of equal length.

d. Parallelograms are rectangles.

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

### MA.5.GR.1.2

Benchmark

MA.5.GR.1.2 Identify and classify three-dimensional figures into categories based on their defining attributes. Figures are limited to right pyramids, right prisms, right circular cylinders, right circular cones and spheres.

Benchmark Clarifications:

*Clarification 1:* Defining attributes include the number and shape of faces, number and shape of bases, whether or not there is an apex, curved or straight edges and curved surfaces or flat faces.

Connecting Benchmarks/Horizontal Alignment

* There are no direct connections outside of this standard; however, teachers are encouraged to find possible indirect connections.

Terms from the K-12 Glossary

* Cone
* Cylinders
* Edge
* Prisms
* Pyramids
* Sphere

Vertical Alignment

|  |  |
| --- | --- |
| **Previous Benchmarks**   * MA.4.GR.1.1 | **Next Benchmarks**  MA.6.GR.2.4 |

Purpose and Instructional Strategies

The purpose of this benchmark is to begin formal categorization of three-dimensional figures based on attributes of their faces, edges and vertices. Three-dimensional figures were identified informally in Kindergarten and Grade 1. The work in Grade 5 prepares students for more detailed work with three-dimensional figures, including finding volumes and surface areas using formulas and nets in Grade 6 (MA.6.GR.2.4).

* Instruction includes having students use language they have already learned and apply it to a larger variety of figures including prisms and pyramids with any number of sides.

Instruction includes explaining that a cone has one flat base, a cylinder has two flat bases and a sphere does not have any flat bases, but each of these figures has a curved surface.

Common Misconceptions or Errors

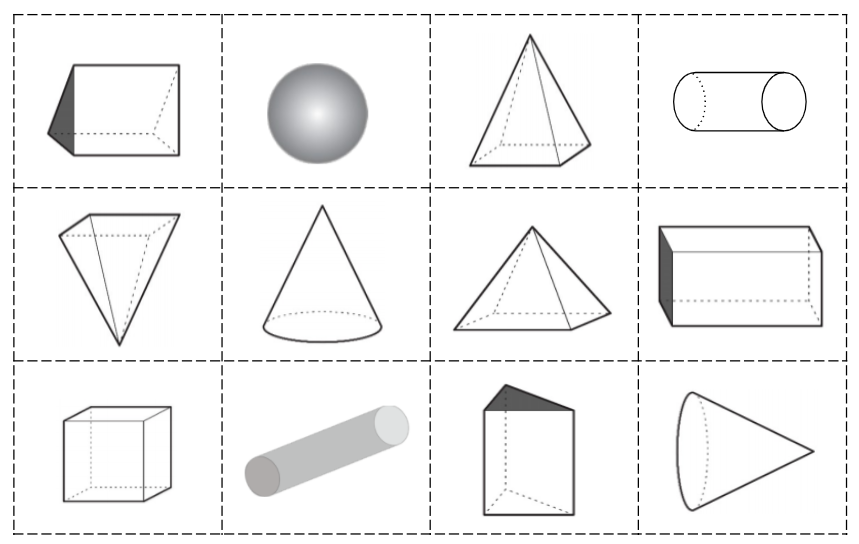
* Students may believe that the orientation of a figure changes the three-dimensional shape.

Strategies to Support Tiered Instruction

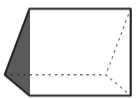
* Instruction may include teacher providing a graphic organizer that contains three-dimensional figure names and definitions from the glossary. Students match images of the figures in different orientations to their definitions.
  + For example, the teacher may provide students with a graphic organizer like the one shown below and a set of three-dimensional figure picture cards. Students match the image to the defining attributes listed.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Figure** | **Pyramid (right, regular)** | **Prism (right)** | **Circular Cylinder (right)** | **Circular Cone** | **Sphere** |
| Defining Attributes | A figure containing a polygonal base and triangular faces. The triangular faces have the same size and shape and they connect the sides of the base to a common point called the apex. | A figure with two parallel bases that are the same shape and size. The bases are connected by rectangular faces that are perpendicular to the bases. A box with identical polygons on each end. | A figure containing two congruent, parallel, circular bases whose edges are connected by a perpendicular curved surface. | A three-dimensional figure with a circular base and an apex that is connected to the base by a collection of line segments that form a curved surface. | A three-dimensional figure with all points equidistant from a point called the center. |
| Examples |  |  |  |  |  |

Example Figure Cards



* Instruction may include providing three-dimensional figures made of plastic or wood and having students identify the shapes that make up their base or bases and faces. Students then look at the definition for each figure and classify it based on the attributes they identified.
  + For example, the teacher may provide the students with a triangular prism like the one shown below. The students then identify the two bases as triangles and the faces connecting them as rectangles. The teacher may provide students with the definitions for three-dimensional figures and have them determine which classification the three-dimensional figure fits in.



Instructional Tasks

*Instructional Task 1 (MTR.4.1)*

Categorize the three-dimensional figures below into the table.

|  |  |  |  |
| --- | --- | --- | --- |
| **Contains circular faces** | **Contains rectangular faces** | **May contain a rectangular face** | **Contains no faces** |
|  |  |  |  |

* Right pyramids
* Spheres
* Right circular cylinders
* Right prisms
* Right circular cones

*Enrichment Task 1*

For a rectangular prism, what shapes are the faces, and which two faces are the figure’s bases?

*Enrichment Task 2*

What do a right triangular pyramid, right square pyramid, and a cone have in common?

Instructional Items

*Instructional Item 1*

Select all the figures that contain an apex.

1. Right pyramids
2. Spheres
3. Right circular cylinders
4. Right prisms
5. Right circular cones

*Enrichment Item 1*

Which figure has the most curved edges?

a. cone

b. cylinder

c. sphere

d. right rectangular prism

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

**MA.5.GR.2***Find the perimeter and area of rectangles with fractional or decimal side lengths.*

### MA.5.GR.2.1

Benchmark

MA.5.GR.2.1 Find the perimeter and area of a rectangle with fractional or decimal side lengths using visual models and formulas..

Benchmark Clarifications:

*Clarification 1:* Instruction includes finding the area of a rectangle with fractional side lengths by tiling it with squares having unit fraction side lengths and showing that the area is the same as would be found by multiplying the side lengths.

*Clarification 2:* Responses include the appropriate units in word form.

Connecting Benchmarks/Horizontal Alignment

* MA.5.NSO.2.3
* MA.5.NSO.2.4
* MA.5.NSO.2.5
* MA.5.FR.2.1
* MA.5.FR.2.2
* MA.5.FR.2.3
* MA.5.AR.1.2
* MA.5.M.1.1

Terms from the K-12 Glossary

* Area Model
* Equation
* Perimeter

Vertical Alignment

|  |  |
| --- | --- |
| **Previous Benchmarks**   * MA.3.GR.2.3 * MA.4.GR.2.1 | **Next Benchmarks**   * MA.6.GR.1.3 |

Purpose and Instructional Strategies

The purpose of this benchmark is for students to understand how to work with fractional and decimal sums and products when calculating perimeter and area. This benchmark connects to previous work where students found areas and perimeters with whole number side lengths in Grade 4 (MA.4.GR.2.1) and prepares for future work of finding area and perimeter on a coordinate plane in Grade 6 (MA.6.GR.1.3).

* During instruction, teachers may encourage students to use models or drawings to assist them with finding the perimeter and area of a rectangle and have them explain how they used the model or drawing to arrive at the solution getting them to understand that multiplying fractional side lengths to find the area is the same as tiling a rectangle with unit squares of the appropriate unit fraction side lengths *(MTR.5.1)*.
* This benchmark provides a natural real-world context and a visual model for the multiplication of fractions and decimals. When finding the area, teachers can begin with students modeling multiplication with whole numbers and progress into the fractional and decimal parts, such as area models using rectangles or squares, fraction strips/bars and sets of counters.
  + For example, ask questions such as, “What does mean?” Then, follow with questions for multiplication with fractions, such as, “What does mean?” “What does mean?” ( sets of ) and “What does mean?” ( of a set of ) *(MTR.2.1, MTR.3.1, MTR.5.1)*.
* Instruction includes providing students with access to the FAST Grade 5 reference sheet throughout instruction so they can practice using the tool to solve real-world volume problems.

Common Misconceptions or Errors

* Students may believe that multiplication always results in a larger number. Working with an area provides them with concrete situations where this is not true.
  + For example, a city block that is mile by mile has an area of of a square mile.
* Students may have difficulty connecting visual models to the symbolic representation using equations. Use concrete visuals to represent problems.

Strategies to Support Tiered Instruction

* Instruction provides opportunities to use concrete visuals to represent problems. Instruction includes providing a rectangle to divide into fractional parts. The teacher provides students with fractional dimensions to divide the figure into to find the area of part of the whole figure. Before calculating the area, students explain if the area will be greater or less than one of the dimensions and explain how they know.
  + For example, the teacher provides students with a blank rectangle and has students divide into fractional parts as shown below. The teacher uses prompts like those shown to help guide the students. After dividing the figure, the students use two different colors to shade the fractional parts and label each side with the shaded dimensions ( or ).

|  |  |
| --- | --- |
| Divide the figure vertically into eights.  Divide the figure horizontally into sixths.  Shade vertically and horizontally. The area of is where the 2 shaded sections overlap.  Is the shaded area greater or less than ? How do you know? | A rectangle divided into forty eight equal parts with eight columns and six rows. Columns one through six, from row one down to row four are colored red. Columns seven and eight from row one down to row four are colored green. Rows five and six from column one over to column six are colored blue. The last four parts that includes rows five and six  and columns seven and eight are white. |

* Instruction includes providing fractional area models printed on transparency sheets. Models include equal size wholes divided into thirds, fourths, fifths, sixths, eighths, tenths, and twelfths. Students use two transparencies to show the area of given dimensions.
  + For example, the teacher asks students to find the area of a figure with side lengths of inch and inch. Students model by shading of one fraction model and of another fraction model. The teacher has students explain if the area will be greater or less than and how they know. The students then overlap the two figures and determine the fractional parts that overlap as being the area.

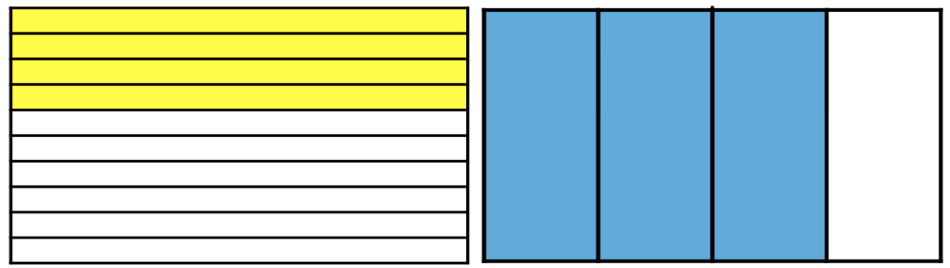
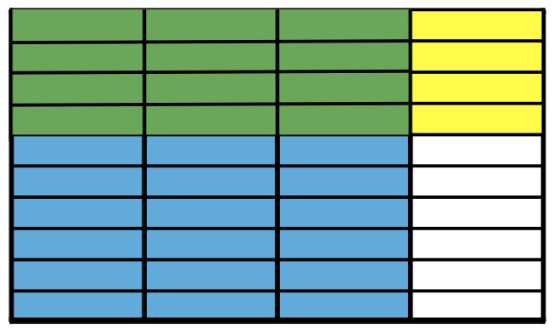


Image showing overlapping , is overlapping.



Instructional Tasks

*Instructional Task 1 (MTR.3.1)*

Margaret draws a rectangle with a length of 5.2 inches. The width of her rectangle is one-half its length.

Part A. Draw Margaret’s rectangle and show its dimensions.

Part B. What is the perimeter of her rectangle in inches?

Part C. What is the area of her rectangle in square inches?

Instructional Items

*Instructional Item 1*

What is the area of the square below?

A square with a side labeled 0.7 inches



\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

**MA.5.GR.3***Solve problems involving the volume of right rectangular prisms.*

### MA.5.GR.3.1

Benchmark

MA.5.GR.3.1 Explore volume as an attribute of three-dimensional figures by packing them with unit cubes without gaps. Find the volume of a right rectangular prism with whole-number side lengths by counting unit cubes.

Benchmark Clarifications:

*Clarification 1:* Instruction emphasizes the conceptual understanding that volume is an attribute that can be measured for a three-dimensional figure. The measurement unit for volume is the volume of a unit cube, which is a cube with edge length of 1 unit.

Connecting Benchmarks/Horizontal Alignment

* MA.5.NSO.2.1

Terms from the K-12 Glossary

* Composite Figure
* Rectangular Prism

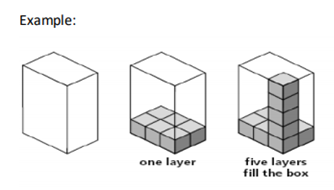
Vertical Alignment

|  |  |
| --- | --- |
| **Previous Benchmarks**   * MA.3.GR.2.1 | **Next Benchmarks**   * MA.6.GR.2.3 |

Purpose and Instructional Strategies

This benchmark introduces volume to students. To introduce the topic, Grade 5 focuses on calculating volume of right rectangular prisms. Students’ prior experiences with volume were restricted to liquid volume (also called capacity). Understanding of volume of right rectangular prisms extends from the understanding of area beginning in Grade 3 (MA.3.GR.2.1), with the idea that one layer (such as the bottom of cube) can be built up by adding more layers of unit cubes. In Grade 6, (MA.6.GR.2.3) students solve volume problems involving rectangular prisms with fraction and decimal side lengths.

* As students develop their understanding of volume, they recognize that a 1unit cube is the standard unit for measuring volume. A unit cube has a length of 1 unit, a width of 1 unit and a height of 1 unit and is referred to as one cubic *unit*. The volume of a three-dimensional figure refers to the number of cubic units that can be placed in it without gaps or overlaps. This cubic unit is written with an exponent of 3 (e.g., , ). Students connect this notation to their understanding of the powers of 10 in our place value system *(MTR.5.1)*.

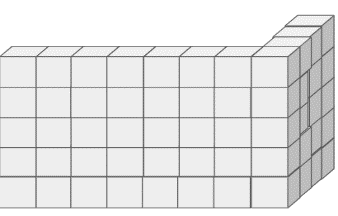


Common Misconceptions or Errors

* Students may incorrectly fill figures to find volume with cubes. Students need to ensure there is no empty space included and that unit cubes are equally sized and packed tightly in without overlaps.

Strategies to Support Tiered Instruction

* Instruction may include providing unit cubes and having students build rectangular prisms with specific dimensions and then calculating the volume.
  + For example, the teacher provides students with unit cubes and the following dimensions: length is 8 units, width is 4 units, and height is 5 units. Students stack equally sized unit cubes and ensure that the cubes are packed tightly with no gaps or overlaps to create a solid three-dimensional figure. Students begin building the figure as shown below, continuing to fill it in until it is complete. Students calculate the volume by multiplying and then decompose the figure and count the cubes to determine if their calculation is correct.



* Instruction may include providing rectangular prisms filled with cubes. Some are filled correctly with no gaps or overlaps, and others have the cubes filling the rectangular prism, but with gaps left between them. Students identify which are stacked correctly to find volume and which are not stacked correctly and record the dimensions of the number of cubes for the height, length, and width, counting the total to determine the volume.

Instructional Tasks

*Instructional Task 1 (MTR.2.1)*

Create as many right rectangular prisms as possible that have exactly 24 cubes. Label the length, width, and height of each prism (in units). How many different prisms can you create? What is the volume (in cubic units) of each prism?

*Enrichment Task 1 (MTR.6.1)*

Molly is putting her cube-shaped blocks into her storage container after she finishes playing with her sister. The storage container is shaped like a right rectangular prism and she has a total of 120 blocks. The bottom layer of her storage container holds exactly 6 rows of 4 blocks each with no gaps or overlaps. The storage container holds exactly 6 layers of blocks with no gaps or overlaps.

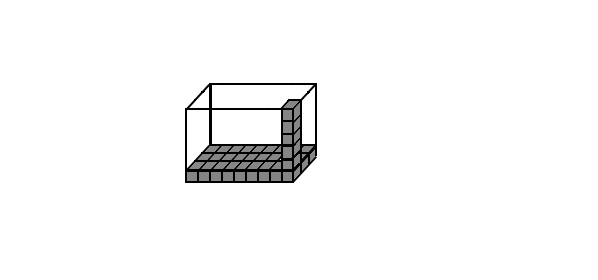
Part A. Will all of Molly’s blocks fit in the storage container? Explain how you know using drawings and equations.

Part B. If there is enough room, determine how many more blocks Molly could fit in the storage container. If there is not enough room, determine how many blocks will not be able to fit in the storage container.

Instructional Items

*Instructional Item 1*

What is the volume of the right rectangular prism?



\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

### MA.5.GR.3.2

Benchmark

MA.5.GR.3.2 Find the volume of a right rectangular prism with whole-number side lengths using a visual model and a formula.

Benchmark Clarifications:

*Clarification 1:* Instruction includes finding the volume of right rectangular prisms by packing the figure with unit cubes, using a visual model or applying a multiplication formula.

*Clarification 2:* Right rectangular prisms cannot exceed two-digit edge lengths and responses include the appropriate units in word form.

Connecting Benchmarks/Horizontal Alignment

* MA.5.NSO.2.1

Terms from the K-12 Glossary

* Composite Figure
* Rectangular Prism
* Volume

Vertical Alignment

|  |  |
| --- | --- |
| **Previous Benchmarks**   * MA.3.GR.2.2 | **Next Benchmarks**   * MA.6.GR.2.3 |

Purpose and Instructional Strategies

The purpose of this benchmark is for students to make connections between packing a right rectangular prism with unit cubes to determine its volume and developing and applying a multiplication formula to calculate it more efficiently. Students have developed experience with area since Grade 3 (MA.3.GR.2.2). For volume, side lengths are limited to whole numbers in Grade 5, and problems extend to fraction and decimal side lengths in Grade 6 (MA.6.GR.2.3).

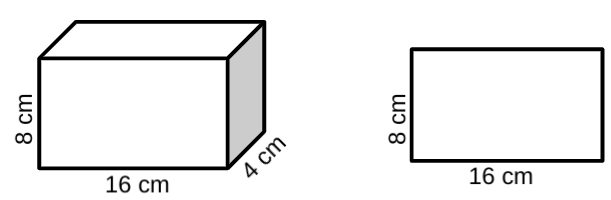
* Instruction may support making connections between the exploration expected of MA.5.GR.3.1 and what is happening mathematically when calculating volume *(MTR.2.1)*.
* Instruction may begin by connecting the measurement of a right rectangular prism to the calculation of a rectangle’s area. The bottom layer of the prism is packed with a number of rows with a number of cubes in each, like area of a rectangle is calculated with unit squares. From there, the third dimension (height) of the prism is calculated by the number of layers stacked atop one another.
* Having students explore how volume is calculated helps students see the patterns and develop a multiplication formula that will help them make sense of the two most common volume formulas, (where represents the area of the rectangular prism’s base) and . If students understand conceptually what the formulas mean, they are more likely to use them effectively and efficiently *(MTR.5.1)*.
* When students use a multiplication formula, it is important for them to see that it is a matter of choice which dimensions of rectangular prisms are named length, width and height. This will help students understand that when calculating the volume of a rectangular prism, the three dimensions are multiplied together and that the order of factors does not matter (commutative property of multiplication).
* Instruction includes providing students with access to the FAST Grade 5 reference sheet throughout instruction so they can practice using the tool to solve real-world volume problems.

Common Misconceptions or Errors

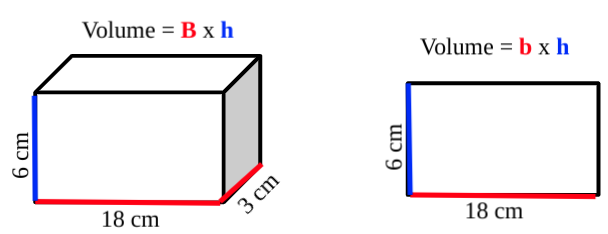
* Students may confuse the difference between *b* in the area formula and in the volume formula . When building understanding of the volume formula for right rectangular prisms, teachers and students should include a visual model to justify their calculations.
* Students may make computational errors when calculating volume. Encourage them to estimate reasonable solutions before calculating and justify their solutions after. Instruction can also encourage students to find efficient ways to use the formula.
  + For example, when calculating the volume of a rectangular prism using the formula , students may find calculating easier if they multiply first, instead of . During class discussions, teachers should encourage students to share their strategies so they can build efficiency.

Strategies to Support Tiered Instruction

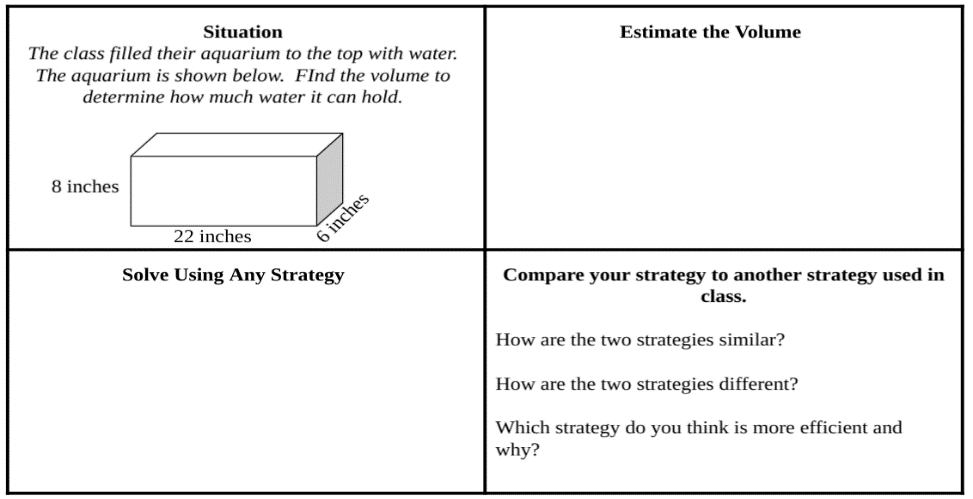
* Instruction may include the use of visual models to justify calculations when using the volume formula for right rectangular prisms.
* Instruction may include differentiating between base in the area formula, and base in the volume formula . The teacher provides students with models of two-dimensional figures, and three-dimensional figures, and has them identify which formula they will use and what the base in each image is.
  + For example, the students highlight the lines included in the base measurement for each figure. Then, they use the base to calculate the area or volume. The teacher provides students with a set of models like the one shown below asking which image they would use the area formula for and which image they would use the volume formula for. Students then highlight the measurements used for the base in the formula. For the first figure, students would use volume and the formula with . For the second figure, students would find area and use the formula with .



* Instruction may include providing models of two-dimensional and three-dimensional figures with the area and volume formula labeled and color-coded with the measurements.
  + For example, the teacher provides students with the following set of visual models and has students explain the difference in the base measurement in each formula. Students calculate the area or volume of each figure using the formula.



* Instruction may include providing a graphic organizer that requires students to estimate the volume of real-world examples provided and then solve using any strategy they would like.
  + For example, the teacher provides students with a graphic organizer similar to the one shown below. Students use it to find the volume of the given example and then compare their strategy to others.



* Instruction may include finding efficient ways to use the formula.
  + For example, when calculating the volume of a rectangular prism using the formula , students may find calculating easier if they multiply (which equals 90) first, instead of . During class discussions, teachers may encourage students to share their strategies so they can build efficiency.
* Instruction may include providing worked examples of volume and having students determine which strategy is the better strategy to use and why.
  + For example, the teacher may provide students with the following image and two examples of how students solved for volume. Student A solved the area of the base first using the Distributive Property to help with the multiplication. Student B used the Associative Property of Multiplication and multiplied first. Students discuss both strategies and explain which would be easier and why.

A rectangular object with a square object in the middle and a square object in the middle



Instructional Tasks

*Instructional Task 1 MTR.5.1)*

Malcolm filled the figure below with cubes. Find the volume of the figure. What would

the volume be if another layer of cubes was added to the top of the figure?

15 cm

8 cm

6 cm

*Enrichment Task 1 (MTR.2.1)*

The Great Graham Cracker Company is looking for a new package design for next year’s boxes. The boxes must be a right rectangular prism and measure 144 cubic centimeters.

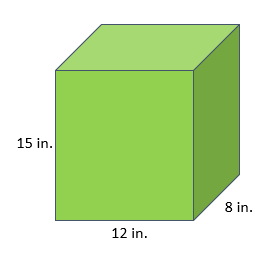
Part A. What are three package designs the company could use? Draw models and write equations to show their volumes.

Part B. Dr. Cruz, the company’s founder, wants the height of the package to be exactly 8 centimeters. What are two package designs that the company can use? Draw models and write equations to show their volumes.

Instructional Items

*Instructional Item 1*

Which of the following equations can be used to calculate the volume of the rectangular prism below?



*Instructional Item 2*

A bedroom shaped like a rectangular prism is 15 feet wide, 32 feet long and measures 10 feet from the floor to the ceiling. What is the volume of the room, in cubic feet?

1. 57 cubic ft.
2. 150 cubic ft.
3. 4,500 cubic ft.
4. 4,800 cubic ft.

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

### MA.5.GR.3.3

Benchmark

MA.5.GR.3.3 Solve real-world problems involving the volume of right rectangular prisms, including problems with an unknown edge length, with whole-number edge lengths using a visual model or a formula. Write an equation with a variable for the unknown to represent the problem.

*Example:* A hydroponic box, which is a rectangular prism, is used to grow a garden in wastewater rather than soil. It has a base of 2 feet by 3 feet. If the volume of the box is 12 cubic feet, what would be the depth of the box?.

Benchmark Clarifications:

*Clarification 1:* Instruction progresses from right rectangular prisms to composite figures composed of right rectangular prisms.

*Clarification 2:* When finding the volume of composite figures composed of right rectangular prisms, recognize volume as additive by adding the volume of non-overlapping parts.

*Clarification 3:* Responses include the appropriate units in word form.

Connecting Benchmarks/Horizontal Alignment

* MA.5.NSO.2.1, MA.5.NOS.2.2
* MA.5.FR.1.1
* MA.5.AR.1.1
* MA.5.M.1.1

Terms from the K-12 Glossary

* Composite Figure
* Rectangular Prism

Vertical Alignment

|  |  |
| --- | --- |
| **Previous Benchmarks**   * MA.4.GR.2.1 | **Next Benchmarks**   * MA.6.GR.2.3 |

Purpose and Instructional Strategies

The purpose of this benchmark is to solve real-world problems involving right rectangular prisms using a visual model or a formula. The real-world problems can require students to find an unknown side length or find the volume of a composite figure (*MTR.7.1*), if the figure can be decomposed into smaller right rectangular prisms. Students are expected to write an equation with a variable for the unknown to represent the problem. Similar expectations for area were developed in Grade 4 (MA.4.GR.2.1) and this work will be extended to include fraction and decimal side lengths in Grade 6 (MA.6.GR.2.3).

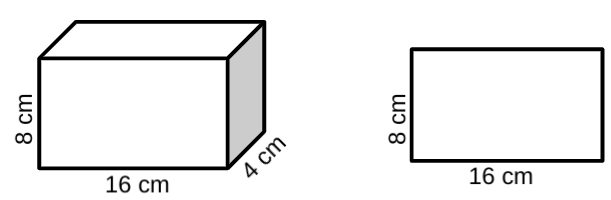
* Instruction of this benchmark can be combined with MA.5.GR.3.2 as students develop and apply understanding of calculating volume of right rectangular prisms using visual models and formulas (*MTR.2.1*).
* While finding volume, teachers may have students communicate and justify their decisions while solving problems (*MTR.4.1*).
* Instruction includes problems with the unknown side length being a fraction (MA.5.FR.1.1).
  + For example, if a box has a base of , and a volume of 20 cubic inches, what is the length of its missing side?
* During instruction teachers may encourage students’ flexibility to use different equations for the same problem.
  + For example, to find the height of a rectangular prism with volume 120 and base dimensions 3 and 10, students can use the any of the follow equations: or or .
* Instruction includes providing students with access to the FAST Grade 5 reference sheet throughout instruction so they can practice using the tool to solve real-world volume problems.

Common Misconceptions or Errors

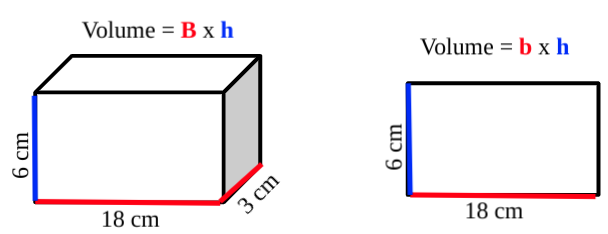
* Students may confuse the difference between b in the area formula and in the volume formula . When building understanding of the volume formula for right rectangular prisms, teachers and students should include a visual model to use to justify their calculations.
* Students may make computational errors when calculating volume. Encourage them to estimate reasonable solutions before calculating and justify their solutions after. Instruction can also encourage students to find efficient ways to use the formula.
  + For example, when calculating the volume of a rectangular prism using the formula, , students may find calculating easier if they first multiply (which equals 90), instead of . During class discussions, teachers should encourage students to share their strategies so they can build efficiency.

Strategies to Support Tiered Instruction

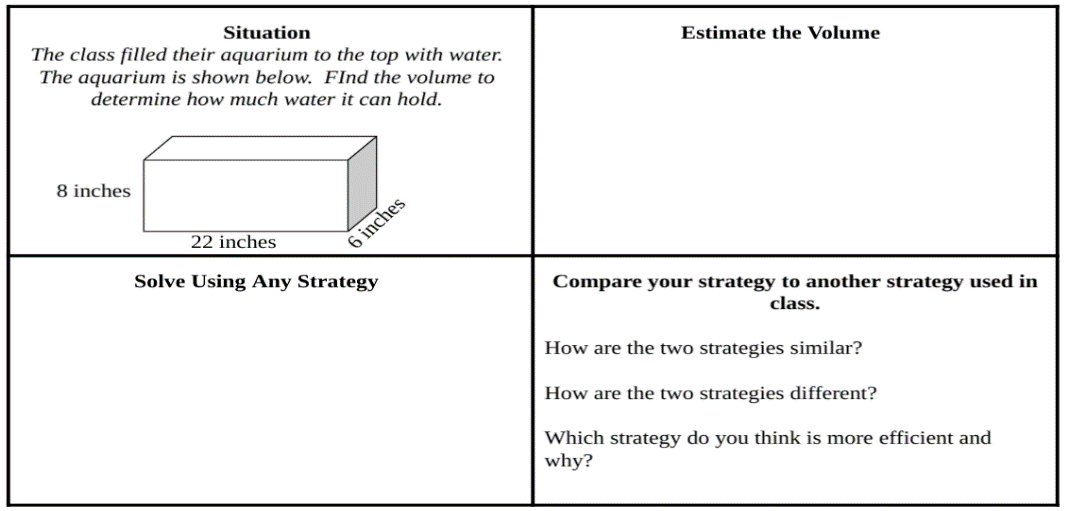
* Instruction includes the use of visual models to justify calculations when using the volume formula for right rectangular prisms.
* Instruction includes differentiating between base in the area formula, and base in the volume formula, . Teacher provides students with models of two-dimensional figures, and three-dimensional figures, and has them identify which formula they will use and what the base in each image is. Students highlight the lines included in the base measurement for each figure and use the base to calculate the area or volume.
  + For example, the teacher provides students with a set of models like the one shown below. The teacher asks students which image they would use the area formula for and which image they would use the volume formula for. Students then highlight the measurements used for the base in the formula. For the first figure, students would use volume and the formula with . For the second figure, students would find area and use the formula with .



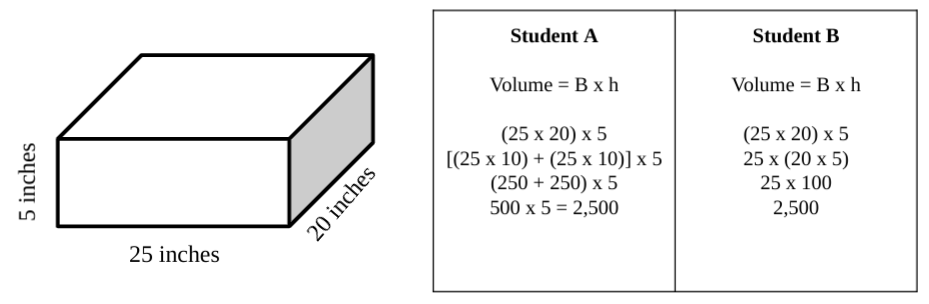
* Instruction includes providing models of two-dimensional and three-dimensional figures with the area and volume formula labeled and color-coded with the measurements.
  + For example, the teacher provides students with the following set of models and has students explain the difference in the base measurement in each formula. Students calculate the area or volume of each figure using the formula.



* Instruction includes providing a graphic organizer that requires students to estimate the volume of real-world examples provided and then solve using any strategy they would like. Students then compare their strategy to the strategies used by other students.
  + For example, the teacher provides students with a graphic organizer similar to the one shown below. Students use it to find the volume of the given example and then compare their strategy to others.



* Instruction includes estimating reasonable solutions before calculating and justifying solutions after. Instruction can also encourage students to find efficient ways to use the formula.
  + For example, when calculating the volume of a rectangular prism using the formula, , students may find calculating easier if they first multiply (which equals 90), instead of . During class discussions, teachers should encourage students to share their strategies so they can build efficiency.
* Instruction includes providing worked examples of volume and having students determine which strategy is the better strategy to use and why.
  + For example, the teacher provides students with the following image and two examples of how students solved for volume. Student A solved the area of the base first using the Distributive Property to help with the multiplication. Student B used the Associative Property of Multiplication and multiplied first. Students discuss both strategies and explain which would be easier and why.



Instruction includes using the 3-Read Protocol to break down the problem. First read the problem to understand the context, possibly without numbers. Then, read to understand the mathematics. Finally, read to create a plan for solving the problem.

Instructional Tasks

*Instructional Task 1 (MTR.6.1)*

The Great Graham Cracker Company places packages of their graham crackers into a larger box for shipping to area grocery stores. Each package of graham crackers is a right rectangular prism that measures 18 cubic inches. The base of each package of graham crackers measures 2 inches by 3 inches. Packages are placed upright into the shipping box.

Part A. If the larger shipping box is a cube with edges that are each 30 inches, how many layers of graham cracker packages can the shipping box hold? Show your thinking using a visual model and equation(s).

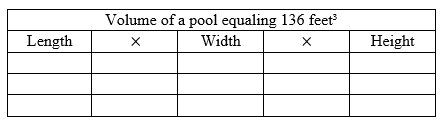
Part B. Will the packages reach the top of the shipping box? If not, what will be the length of the gap from the top of the package to the top of the shipping box?

Part C. How many graham cracker packages will fit in the shipping box?

*Instructional Task 2*

The volume of Kara’s swimming pool is 256 cubic feet.

What are the possible dimensions of her pool?



*Instructional Task 3*

Select all the possible dimensions of an aquarium with a volume of 216 cubic feet.

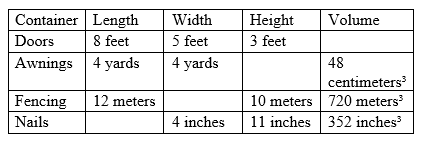
1. Length-3 feet, width-9 feet, height-8 feet
2. Length-4 feet, width-9 feet, height-6 feet
3. Length-4 feet, width-9 feet, height 16 feet
4. Length-3 feet, width-8 feet, height-8 feet
5. Length-6 feet, width-3 feet, height-12 feet

*Instructional Task 4*

The chart below shows a variety of containers for different items.

Some dimensions are missing in the chart below.

Find the missing dimensions for each.



*Instructional Task 5*

For a holiday event, Flamingo Elementary School is creating a snowman out of three stacked boxes shaped like cubes. When stacked, the snowman will be a composite three-dimensional figure.

* The edge length of the bottom box is 13 inches.
* The edge length of the middle box is 2 inches less than the bottom box.
* The edge length of the top box is 2 inches less than the middle box.

Part A. Draw a model to show the composite figure.

Part B. What is the total volume of the snowman?

Part C. Explain why the snowman is considered a composite three-dimensional figure.

Instructional Items

*Instructional Item 1*

Select all of the following that could be the dimensions of the base of a rectangular box with height of 16 inches and volume of 128 cubic inches.

1. 2 inches x 4 inches
2. 3 inches x 3 inches
3. 1 inch x 8 inches
4. 4 inches x 2 inches
5. 56 inches x 56 inches

*Instructional Item 2*

Mia is measuring the inside of her desk.

The area of the base is 88 square centimeters.

The height is 16 centimeters.

What is the volume of the inside of the desk, in cubic centimeters?

*Instructional Item 3*

A flower box for roses is in the shape of a rectangular prism.

The box is 6 inches tall, 9 inches wide, and has a volume of 918 cubic inches.

What is the length of the box, in inches?

*Instructional Item 4*

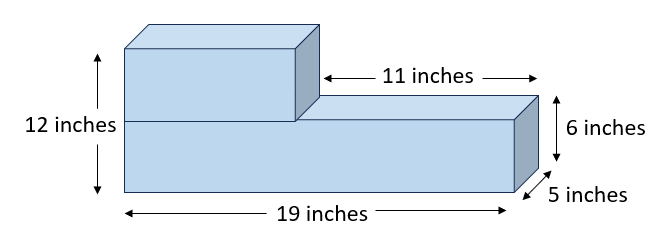
Mark stores his fishing lures in a cube-shaped box.

The cube has a volume of 64 cubic inches.

What are the lengths of the edges of the cube?

*Instructional Item 5*

The figure below is a composite right rectangular prism.



Find the volume of the composite figure, in cubic inches.

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

**MA.5.GR.4***Plot points and represent problems on the coordinate plane.*

### MA.5.GR.4.1

Benchmark

MA.5.GR.4.1 Identify the origin and axes in the coordinate system. Plot and label ordered pairs in the first quadrant of the coordinate plane.

Benchmark Clarifications:

*Clarification 1:* Instruction includes the connection between two-column tables and coordinates on a coordinate plane.

*Clarification 2:* Instruction focuses on the connection of the number line to the - and -axis. *Clarification 3:* Coordinate planes include axes scaled by whole numbers. Ordered pairs contain only whole numbers.

Connecting Benchmarks/Horizontal Alignment

* MA.5.AR.3.2
* MA.5.DP.1.1

Terms from the K-12 Glossary

* Coordinate Plane (first quadrant)
* Origin
* -axis
* -axis

Vertical Alignment

|  |  |
| --- | --- |
| **Previous Benchmarks**   * MA.4.NSO.1.3 | **Next Benchmarks**   * MA.6.GR.1 |

Purpose and Instructional Strategies

The purpose of this benchmark is for students to extend their thinking from Grade 4 (MA.4.NSO.1.3) about horizontal and vertical number lines to plot and label whole number ordered pairs on a coordinate plane. In addition, students will make a connection between a two-column table and the ordered pairs represented on the coordinate plane. In Grade 6 (MA.6.GR.1.1), students plot rational number pairs in all four quadrants of the coordinate plane.

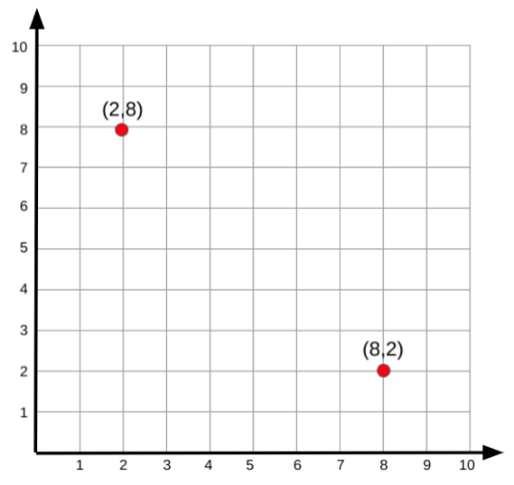
* Instruction includes relating the coordinate plane as the intersection of two axes – a horizontal number line called the -axis and a vertical number line called the -axis. The number lines that form the axes are perpendicular and meet at the origin, labeled by the ordered pair (*MTR.5.1*).
* When students learn to plot ordered pairs represented in a two-column table, they should understand that the ordered pair represents how far to travel from the origin along the - and -axes.
  + For example, students should understand that in the ordered pair , the point travels along the -axis 2 whole units to the right, and then vertically (parallel to the -axis) 4 units up (*MTR.5.1*).

Common Misconceptions or Errors

* Students can confuse the - and -values in an ordered pair and move vertically along the -axis before moving horizontally along the -axis.
  + For example, they may mean to plot and label the ordered pair , but plot and label instead. To assist students with this misconception, have students practice with creating directions for their student peers to follow to allow them to gain a better understanding of the direction and distance on the coordinate plane.
* Some students may not understand what an - or -coordinate value of 0 represents. During instruction, students should justify why ordered pairs with a 0 will plot on the -axis or -axis.

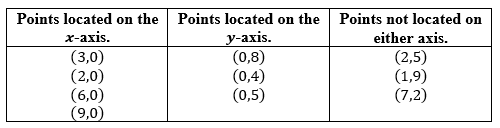
Strategies to Support Tiered Instruction

* Instruction includes the teacher providing coordinate points to graph in quadrant 1 of the coordinate plane along with two small objects. The students explain how they move the object along the -axis and then up the -axis to the location provided. The teacher then provides the points reversed to graph and has students explain the difference in how they move the second object compared to the first.
  + For example, the teacher provides students with a coordinate plane like the one shown below. The teacher provides a set of coordinate points such as . Students take turns moving an object, such as a two-colored counter, and explain the location of the point using the - and -axis in their explanation. The teacher then provides the points in reverse, . The next student will move a second object and explain the location of the point as well as the difference between the two locations.



* Instruction includes the teacher providing a set of cards that have coordinate points on them, some with 0 as the location on the -axis, some with 0 as the location on the -axis, others with no 0 in the coordinates. Students sort the cards into three categories: points located on the -axis, points located on the -axis and neither. Students will justify their reasoning by explaining how the 0, or lack of a 0, in each set of points helped them.
  + For example, the teacher provides cards with the following points on them:

Students sort the points into three categories as shown below.



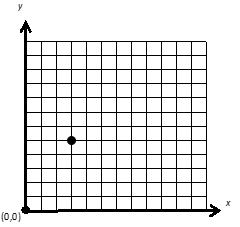
* Instruction includes the teacher creating a giant coordinate plane on the floor with painters' tape or outside with sidewalk chalk. The teacher or a student will then create directions for their peers to follow. The teacher or student will provide a set of coordinate points, including those with 0 as the - or -coordinate. Another student will physically move to the location, describing as they move, which axis they are moving on and counting the spaces until they reach their final location.
  + For example, the teacher or a student tells a student to move to the location of on the coordinate plane. The student says, “I begin at the origin which is and move 1, 2, 3, 4 spaces to the right on the -axis. I then move 1, 2, 3, 4, 5, 6 spaces up on the -axis to my final location of .”
* For example: The teacher provides a student with the location . The student will move along the -axis 5 spaces and stop. The teacher provides another student with the location . That student moves up the -axis 5 spaces and stop. The teacher will then have the students explain how their location ended up on the - or -axis as well as the relationship between those located on the -axis and those located on the -axis.

Instructional Tasks

*Instructional Task 1 (MTR.3.1)*

Part A. A point has coordinates . If you were to graph this point on a coordinate plane, what does the 3 tell you to do?

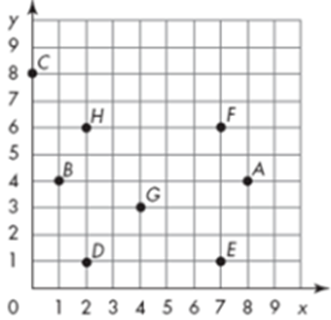
Part B. Consider the same point with coordinates . What does the 5 tell you to do?



Part C. The point above has coordinates . Which of these is the -coordinate? Which of these is the -coordinate?

*Instructional Task 2*

Use the coordinate plane below to answer the questions.



Billy is walking around town.

1. His house is at (7,1). What letter represents Billy’s house?
2. He is meeting his friends at the skate park. To get to the skate part, he moves 5 left and 5 up. What letter represents the skate park?
3. On his way to the skate park, he walked through the gas station parking lot. What letter represents the gas station?
4. After the skate park, Billy went to the store to get a drink. He moved 1 left and 2 down. What letter represents the school?
5. After the store, Billy went to the football field. He moved 6 right and 2 up. What letter represents the football field?
6. He goes to a friend’s house for lunch. His friend lives close to the football field. Which letter is closest to the football field?
7. After lunch, Billy goes to the movies with his friends. He moves 8 left and 4 up. What letter represents the movie theater?
8. On his way home, Billy stops to visit his gramma. Her house is at (4,3). What letter represents gramma’s house? How does Billy have to move from the movie theater to get to gramma’s house?
9. Finally, Billy goes home. How does he move to go home?

*Instructional Task 3*

Roll 2 number cubes to create an ordered pair.

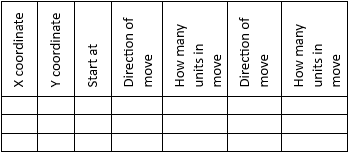
Fill in the table using the numbers generated with the number cubes.

Variation 1:

Start from the origin (0,0) for each roll.

Variation 2:

With each roll, start at the last coordinate rolled and find the new coordinate.



*Instructional Task 4*

Using the coordinate plane below, create an assortment of triangles and rectangles.

1. Identify the shape created.
2. Identify the ordered pairs for each vertex of the shape.

Instructional Items

*Instructional Item 1*

What ordered pair represents the origin of a coordinate plane?

*Instructional Item 2*

A point has coordinates . If you were to plot this point on a coordinate plane, what does the 1 tell you to do?

* 1. From the origin, move along the -axis 1 unit up.
  2. From the origin, move along the -axis 1 unit up.
  3. From the origin, move along the -axis 1 unit right.
  4. From the origin, move along the -axis 1 unit right.

*Instructional Item 3*

A point has the coordinates (4, 6).

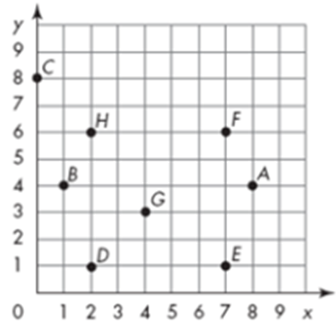
To get to the coordinate,

* Where do you start?
* How do you move?
* How many units do you move?
* Do you move again? If yes, which direction and how many units?

*Instructional Item 4*

Joerg is plotting a square on the coordinate plane below.

Which four ordered pairs would he identify in his square?



\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

### MA.5.GR.4.2

Benchmark

MA.5.GR.4.2 Represent mathematical and real-world problems by plotting points in the first quadrant of the coordinate plane and interpret coordinate values of points in the context of the situation.

*Example:* For Kevin’s science fair project, he is growing plants with different soils. He plotted the point ) for one of his plants to indicate that the plant grew inches by the end of week .

Benchmark Clarifications:

*Clarification 1:* Coordinate planes include axes scaled by whole numbers. Ordered pairs contain only whole numbers.

Connecting Benchmarks/Horizontal Alignment

* MA.5.AR.1.1
* MA.5.AR.3.2
* MA.5.DP.1.1

Terms from the K-12 Glossary

* Coordinate Plane (first quadrant)
* Origin
* -axis
* -axis

Vertical Alignment

|  |  |
| --- | --- |
| **Previous Benchmarks**  MA.4.NSO.1.3 | **Next Benchmarks**   * MA.6.GR.1 |

Purpose and Instructional Strategies

The purpose of this benchmark is for students to interpret coordinate values plotted in mathematical and real-world contexts. Students have been plotting and interpreting numbers on a number line since Kindergarten. Students’ first experience with interpreting points plotted on a coordinate plane is in Grade 5, which leads to the foundational understanding needed throughout middle school.

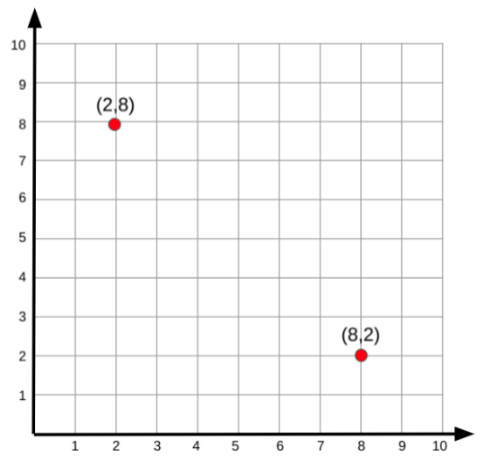
* An example of interpreting coordinate values of points in a mathematical context could be identifying points of a rectangle plotted on the coordinate plane.
* An example of interpreting coordinate values of points in a real-world context could look like the example in the benchmark description. In this real-world example, students would interpret that each axis represents a variable describing a situation. The -axis represents number of weeks and the -axis represents plants’ heights in inches.
* During instruction, teachers should provide plenty of opportunities for students to both plot and interpret ordered pairs on a coordinate plane. Teachers may connect the expectations of this benchmark with MA.5.GR.4.1 by having students represent the points plotted on two-column tables as well *(MTR.4.1, MTR.7.1)*.
* In real-world contexts teachers may allow students the flexibility to decide which variable is represented by and which is represented by . Students may be encouraged to explain their preference.
* During instruction, students should be given the flexibility to decide how to scale their graphs for a given real-world context. Students may be encouraged to explain their preference.

Common Misconceptions or Errors

* Students can confuse the - and -values in an ordered pair and move vertically along the -axis before moving horizontally along the -axis.
  + For example, they may mean to plot and label the ordered pair , but plot and label instead.
* Some students may not understand what an - or -coordinate value of 0 represents. During instruction, students should justify why ordered pairs with a 0 will plot on the -axis or -axis.

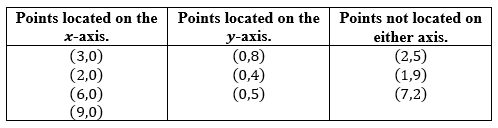
Strategies to Support Tiered Instruction

* Instruction includes the teacher providing coordinate points to graph in quadrant 1 of the coordinate plane along with two small objects. The students explain how they move the object along the -axis and then up the -axis to the location provided. The teacher then provides the points reversed to graph and has students explain the difference in how they move the second object compared to the first.
  + For example, the teacher may provide students with a coordinate plane like the one shown below. The teacher provides a set of coordinate points such as . Students take turns moving an object, such as a two-colored counter, and explain the location of the point using the - and -axis in their explanation. The teacher will then provide the points in reverse, . Students will move a second object and explain the location of the point as well as the difference between the two locations.



* Instruction includes the teacher providing a set of cards that have coordinate points on them, some with 0 as the location on the -axis, some with 0 as the location on the -axis, others with no 0 in the coordinates. Students sort the cards into three categories: points located on the -axis, points located on the -axis and neither. Students justify their reasoning by explaining how the 0, or lack of a 0, in each set of points helped them.
  + For example, the teacher provides cards with the following points on them:

Students sort the points into three categories as shown below.



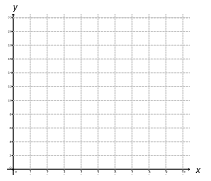
* Instruction includes the teacher creating a giant coordinate plane on the floor with painters' tape or outside with sidewalk chalk. The teacher or a student will then create directions for their peers to follow. The teacher or student provides a set of coordinate points, including those with 0 as the - or -coordinate. Another student physically moves to the location, describing as they move, which axis they are moving on and counting the spaces until they reach their final location.
  + For example, the teacher or a student tells another student to move to the location of on the coordinate plane. The student says, “I begin at the origin which is and move 1, 2, 3, 4 spaces to the right on the -axis. I then move 1, 2, 3, 4, 5, 6 spaces up on the -axis to my final location of .”
  + For example, the teacher provides a student with the location . The student moves along the -axis 5 spaces and stop. The teacher provides another student with the location . That student moves up the -axis 5 spaces and stop. The teacher then has students explain how their location ended up on the - or -axis as well as the relationship between those located on the -axis and those located on the -axis.
* Instruction includes using the 3-Read Protocol to break down the problem. First read the problem to understand the context, possibly without the numbers. Then, read to understand the mathematics. Finally, read to create a plan for solving the problem.

Instructional Tasks

*Instructional Task 1 (MTR.7.1)*

Lukas can make four bracelets per hour and he will work for five hours. Make a two-column table where the first column contains the numbers 1, 2, 3, 4, 5 indicating the number of hours worked, and the second column shows how many total bracelets he has made in that many hours.

Plot points on the coordinate plane to represent your table, where the *-*coordinate represents the number of hours worked and the -coordinate represents the number of bracelets made.

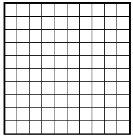


*Instructional Task 2*

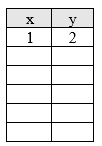
Using the rule , find the next five ordered pairs.

Make sure to label both the x- and y-axis.

Mark and label all the ordered pairs on the coordinate grid.



Rule:

**

*Instructional Task 3*

Pat is planning the set up for his new playroom.

He is using a coordinate grid.

He has a rectangular table that he has begun to mark on the coordinate grid.

He has plotted (10,4), (12,4), and (10,8).

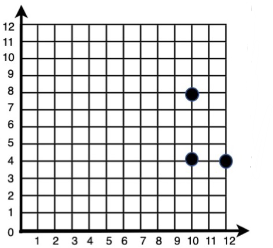
What is the final coordinate pair for his table?

He has a miniature train track that is 12 units long and 5 units wide.

* What are the coordinate pairs for the only space he would put the miniature train track?
* Label the points on the coordinate grid.

He has a 2-unit by 2-unit shelf he wants to put in a corner.

* List the coordinate pairs for all the possible locations.
* Mark the points and label them on the coordinate grid.



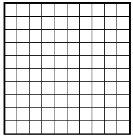
*Instructional Task 4*

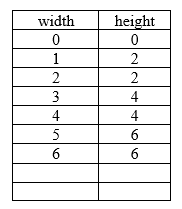
Label the x- and y- axis.

Mel is growing a tomato plant.

Her measurements are in the table below.

* Plot the ordered pairs for the growth of the plant.
* Based on the pattern of growth, predict the next two ordered pairs.

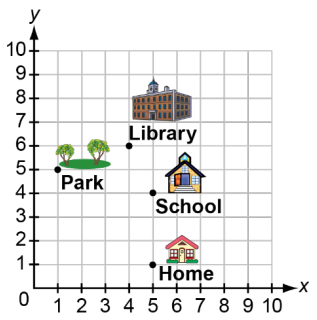
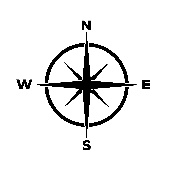


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Instructional Items

*Instructional Item 1*

The map below shows the location of several places in a town.

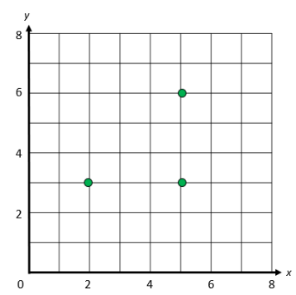


The fire department is 2 blocks north of the library. What ordered pair represents the location of the fire department?

1. (4, 2)
2. (2, 4)
3. (4, 8)
4. (8. 4)

*Instructional Item 2*

Deanna is plotting a square on the coordinate plane below.

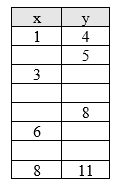


What ordered pair would represent the fourth vertex?

* 1. (6, 2)
  2. (2, 6)
  3. (2, 0)
  4. (0, 2)

*Instructional Item 3*

* If the Rule is x + 3 = y, fill in the missing numbers on the table.
* Plot all the ordered pairs onto the coordinate grid.



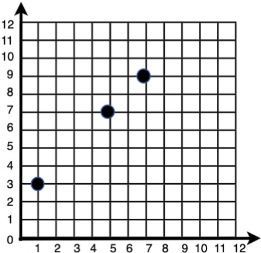


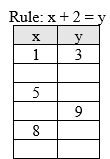
*Instructional Item 4*

Mr. Kelsey is planning to build a fence.

He is using a coordinate grid to determine how many posts he needs to buy.

* + Use the rule to find the ordered pairs.
  + Mark all the points on the coordinate grid.
  + How many posts does Mr. Kelsey need to buy?





\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

## Data Analysis & Probability

**MA.4.DP.1** *Collect, represent and interpret data and find the mode, median and range of a data set.*

### MA.4.DP.1.1 & MA.5.DP.1.1

Benchmark (Grade 4)

MA.4.DP.1.1 Collect and represent numerical data, including fractional values, using tables, stem-and-leaf plots or line plots.

*Example:* A softball team is measuring their hat size. Each player measures the distance around their head to the nearest half inch. The data is collected and represented on a line plot

Benchmark Clarifications:

*Clarification 1:* Denominators are limited to 2, 3, 4, 5, 6, 8, 10, 12, 16 and 100.

Benchmark (Grade 5)

MA.5.DP.1.1 Collect and represent numerical data, including fractional and decimal values, using tables, line graphs or line plots.

*Example:* Gloria is keeping track of her money every week. She starts with , after one week she has , after two weeks she has and after three weeks she has . Represent the amount of money she has using a line graph.

Benchmark Clarifications:

*Clarification 1:* Denominators are limited to 2, 3, 4, 5, 6, 8, 10, 12, 16 and 100.

Connecting Benchmarks/Horizontal Alignment

* MA.4.NSO.1.5
* MA.4.FR.1.3/1.4
* MA.4.M.1.1
* MA.5.NSO.1.4
* MA.5.AR.1.2
* MA.5.GR.4.1/4.2

Terms from the K-12 Glossary

* Line Graphs
* Line Plot
* Stem-and-Leaf Plot

Vertical Alignment

|  |  |
| --- | --- |
| **Previous Benchmarks**   * MA.3.DP.1.1 | **Next Benchmarks**   * MA.6.DP.1.5 |

Purpose and Instructional Strategies

The purpose of this benchmark is to collect and display authentic numerical data in tables, stem-and-leaf plots, line graphs or line plots, including fractional and decimal values. This concept builds on collecting and displaying whole number data using line plots, bar graphs, and tables in the Grade 3 Accelerated course (MA.3.DP.1.1). In grade 6, this work will extend to box plots and histograms (MA.6.DP.1.5)

* A stem-and-leaf plot displays numerical data and uses place value to display data frequencies. In a stem-and-leaf-plot, a number is decomposed so that leaves represent the smallest part of a number (e.g., ones, fractions less than 1) and the stem consists of all its other place values (e.g., hundreds, tens, ones in fractions greater than 1).
* A stem-and-leaf plot organizes data by size (e.g., least to greatest or greatest to least). Stem-and-leaf plots help students build line plots. Stem-and-leaf plots can help students identify benchmarks for their number lines when creating a line plot.
* Instruction of line plots should first focus on creating appropriate number lines that allow a data set to be displayed.
* Measurement data can be gathered (including measuring with precision to the nearest inch) and displayed on tables, line plots, stem-and leaf-plots and line graphs.
* Instruction with line graphs should develop the understanding that values in this graph often represent data that changes over time.

Instruction should include identifying the meaning of the points presented on the -axis and -axis with both axes being labeled correctly.

Common Misconceptions or Errors

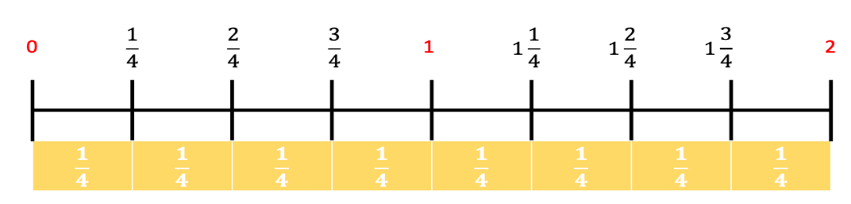
* For line plots, students may misread a number line and have difficulty because they use whole-number names when counting fractional parts on a number line instead of the fraction name. Students also count the tick marks on the number line to determine the fraction, rather than looking at the “distance” or “space” between the marks.
* For stem-and-leaf plots, students may read they key incorrectly. Some students may try to represent numerical data in a stem-and-leaf plot without first arranging the leaves for each stem in order

Strategies to Support Tiered Instruction

* Instruction includes opportunities to use concrete models and draw number lines to connect learning with fraction understanding. Students plot fourths on the number line, paying particular attention to what each tick mark and the “distance” between each tick mark represents.
  + Example:

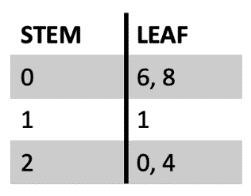


* + For example, utilizing fraction strips or tiles, students will be able to connect fractional parts to the measurement on a number line.



* Instruction includes representing numerical data in a stem and leaf plot and ordering the data from least to greatest. The stem will be the greatest place value of the largest number in the set. With a set of mixed numbers, the stems will be the whole numbers.
  + For example, create a steam and leaf plot using the data set shown.

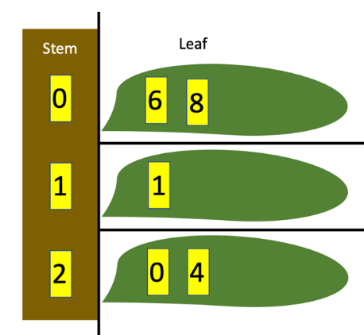
Data set: 6, 8, 11, 20, 24



* Instruction includes representing numerical data in a stem-and-leaf plot and writing the data set on index cards or sticky notes.
  + Example:



After organizing the data set in order from least to greatest, the students rip each number, separating the place values and place them on the graphic organizer. The greater place value will be the stem, the tens place for this example. The lesser place value will be the leaf, the ones place in this example. Since the stems will only be labeled once, the numbers with the same place value will be stacked on top of each other. Each of the leaves will be represented, even if repeated. Numbers with 0 in the tens place will be represented by a 0 for the stem.



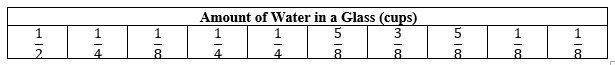
Instructional Tasks

*Instructional Task 1*

Measure the length of 10 used pencils in the class to the nearest inch. Create a stem-and-leaf plot and a line plot to represent the lengths of all ten pencils.

*Instructional Task 2 (MTR.3.1)*

Claire studied the amount of water in different glasses. The data she collected is below. Use her data to create a line plot to show the amount of water in the glasses.



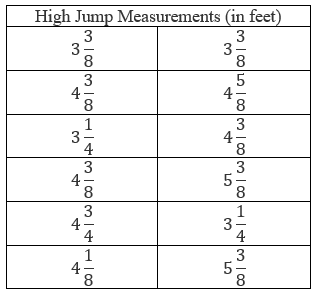
Instructional Items

*Instructional Item 1*

Laura was given the data in the chart below.

Part A. What is the approximate change in the kitten’s mass, in grams, between Days 3 and 4?

Part B. What is the approximate change in the kitten’s mass, in grams, between Days 2 and 5?

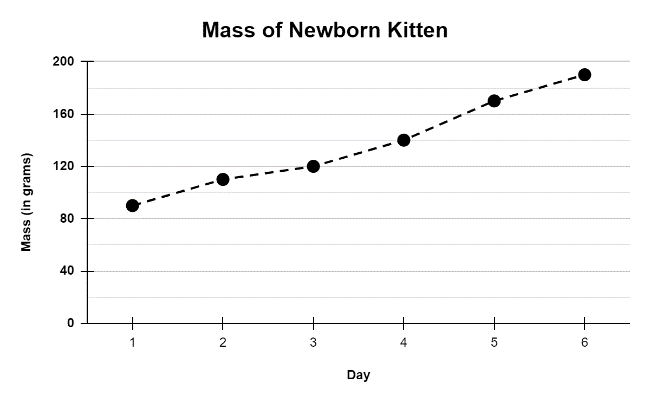


She was asked to create a line plot to represent her data. How many **X’s** will she place above ?

1. 3
2. 4
3. 8
4. 12

*Instructional Item 2*

A line graph is shown.



Part A. What is the approximate change in the kitten’s mass, in grams, between Days 3 and 4?

Part B. What is the approximate change in the kitten’s mass, in grams, between Days 2 and 5?

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

### MA.4.DP.1.2 & MA.5.DP.1.2

Benchmark (Grade 4)

MA.4.DP.1.2 Determine the mode, median or range to interpret numerical data including fractional values, represented with tables, stem-and-leaf plots or line plots.

*Example:* Given the data of the softball team’s hat size represented on a line plot, determine the most common size and the difference between the largest and the smallest sizes.

Benchmark Clarifications:

*Clarification 1:* Instruction includes interpreting data within a real-world context.

*Clarification 2:* Instruction includes recognizing that data sets can have one mode, no mode or more than one mode.

*Clarification 3:* Within this benchmark, data sets are limited to an odd number when calculating the median.

*Clarification 4:* Denominators are limited to 2, 3, 4, 5, 6, 8, 10, 12, 16 and 100.

Benchmark (Grade 5)

MA.5.DP.1.2 Interpret numerical data, with whole-number values, represented with tables or line plots by determining the mean, mode, median or range.

*Example:* Rain was collected and measured daily to the nearest inch for the past week. The recorded amounts are and . The range is inches, the modes are 0 and inches, and the mean value can be determined as , which is equivalent to of an inch. This mean would be the same if it rained of an inch each day.

Benchmark Clarifications:

*Clarification 1:* Instruction includes interpreting the mean in real-world problems as a leveling out, a balance point or an equal share.

Connecting Benchmarks/Horizontal Alignment

* MA.4.FR.1.3/1.4
* MA.5.FR.1.1
* MA.5.AR.1.1

Terms from the K-12 Glossary

* Line Plots
* Mean
* Median
* Mode
* Range

Stem-and-Leaf Plot

Vertical Alignment

|  |  |
| --- | --- |
| **Previous Benchmarks**   * MA.3.DP.1.2 | **Next Benchmarks**   * MA.6.DP.1.2 * MA.6.DP.1.6 |

Purpose and Instructional Strategies

The purpose of this benchmark is to interpret numerical data by using the mean, mode, median and range as measures of center and spread in a set of data. In Grade 6, a focus will be on comparing the advantages and disadvantages of the mean and median.

* Instruction includes providing students multiple opportunities to organize their data (MA.4.FR.1.4). When finding median and mode, it is important for students to organize their data, putting it in order from least to greatest.
* With the data organized, students can determine:
  + range by subtracting the least value from the greatest value in the set.
  + mode by finding the value that occurs most often.
  + median by finding the value in middle of the set.
  + mean by finding the average of the set of numbers.
  + For example, fifteen students were asked to rate how much they like fourth grade on a scale from one to ten. Here is the data collected: 1, 10, 9, 6, 5, 10, 9, 8, 3, 3, 8, 9, 7, 4, 5. The first step is to put the data in ascending order.
  + 1, 3, 3, 4, 5, 5, 6, 7, 8, 8, 9, 9, 9, 10, 10. The median is 7, the mode is 9 and the range is 9.

Common Misconceptions or Errors

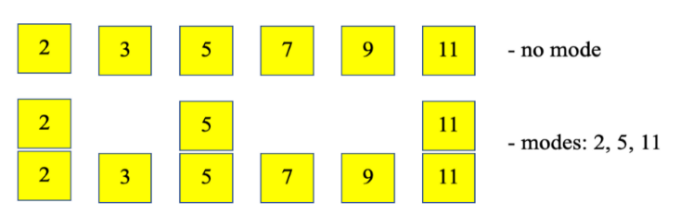
* Students sometimes have difficulty understanding that there may be no mode or more than one mode of a data set. Examples should be given to explicitly teach this concept.
* Students may confuse the range with the number of data points.
* Students may confuse the mean and median of a data set. During instruction, teachers should provide students with examples where the median and mean of a data set are not close in value.

Strategies to Support Tiered Instruction

* Instruction includes providing a data set that may have no mode or more than one mode.
* For example, for the data set 2, 3, 5, 7, 9, 11, there is no mode.
  + - For example, for the data set 2, 2, 3, 5, 5, 7, 9, 11, 11, the modes are 2, 5, and 11.
* Instruction includes providing the data set on index cards or sticky notes. Students can then easily arrange the data in order from least to greatest. This will assist in finding the median of the data set.
* Instruction includes examples where the mean and the median are not close in value and uses a data set to explain the difference between mean and median.
  + For example, students use the data shown to explain the difference between mean (which is 7) and median (which is 4) and to model how the mean is calculated

and how the median is found.

* For numbers that repeat, students stack the numbers on top of each other. This helps with understanding if there is no mode, or more than one mode.
  + For example, this visual helps students find the mode and see when there is no mode or more than one mode.



* Instruction includes showing how to cover up the data points in the middle of the line plot so that only the first and last data points are shown. This allows students to focus on the values that will be used to calculate the range. Students subtract the least value on the line plot with an X from the greatest value with an X

|  |  |  |
| --- | --- | --- |
| Line plot for numbers above. |  | Line plot with some data concealed. |

Instructional Tasks

*Instructional Task 1 ((MTR.7.1)*

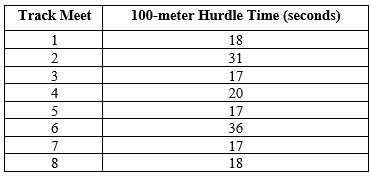
Measure the length of 10 used pencils in the class to the nearest inch.

Part A. Create a stem-and-leaf plot and a line plot to represent the length of all ten pencils.

Part B. Use your completed line plot to find the median, range, mode and mean of the data.

*Instructional Task 2 (MTR.7.1)*

Bobbie is a fourth grader who competes in the 100-meter hurdles. In her 8 track meets during the season, she recorded the following times to the nearest second.



Part A. What is the mean time, in seconds, of Bobbie’s 100-meter hurdles?

Part B. What is the median time, in seconds, of Bobbie’s 100-meter hurdles?

Part C. What is the mode time, in seconds, of Bobbie’s 100-meter hurdles?

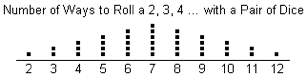
Part D. If you were Bobbie, which of these results would you report to your friend?

Instructional Items

*Instructional Item 1*

The line plot below shows all of the results of the sum of two six-sided dice.

**Number of Ways to Roll a 2, 3, 4 … with a Pair of Dice**

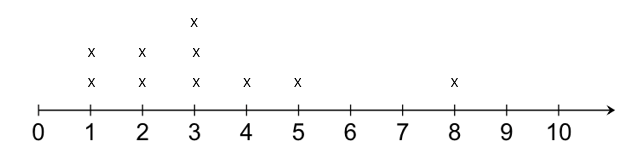


What is the mode of the data on the line plot?

1. 12
2. 10
3. 7
4. 6

*Instructional Item 2*

There was a pie-eating contest at the county fair. The line plot below shows the number of pies each of the 10 contestants ate. Use the line plot to determine the mean, mode, median and range of the data.



\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

### MA.4.DP.1.3

Benchmark

MA.4.DP.1.3 Solve real-world problems involving numerical data.

*Example:* Given the data of the softball team’s hat size represented on a line plot, determine the fraction of the team that has a head size smaller than 20 inches.

Benchmark Clarifications:

*Clarification 1:* Instruction includes using any of the four operations to solve problems.

*Clarification 2:* Data involving fractions with like denominators are limited to 2, 3, 4, 5, 6, 8, 10, 12, 16 and 100. Fractions can be greater than one.

*Clarification 3:* Data involving decimals are limited to hundredths.

Connecting Benchmarks/Horizontal Alignment

* MA.4.NSO.1.5
* MA.4.NSO.2.7
* MA.4.AR.1.2
* MA.4.AR.1.3

Terms from the K-12 Glossary

Numerical Data

Vertical Alignment

|  |  |
| --- | --- |
| **Previous Benchmarks**   * MA.3.DP.1.2 | **Next Benchmarks**   * MA.5.DP.1.2 |

Purpose and Instructional Strategies

The purpose of this benchmark is to use data sets as real-world context for doing arithmetic with whole numbers, fractions and decimals beyond finding measures of center and spread.

* Instruction includes having students solve one- and two-step problems from a given data set or by comparing two data sets in the same units.
* Instruction includes problems that involve addition, subtraction, multiplication or division.
* This benchmark should be taught with MA.4.DP.1.1 and MA.4.DP.1.2 (collecting and representing data). Students should have a strong command of creating and interpreting line plots and stem-and-leaf plots to be successful with the interpretation these data displays.

Common Misconceptions or Errors

* Students can make errors when writing equations used to solve problems with numerical data. During instruction, expect students to justify their equations and solutions.

Strategies to Support Tiered Instruction

* Instruction includes visualizing word problems. The Three-Reads Protocol is a strategy to help students conceptualize what the question is asking. Students draw pictures or models to represent what is happening in the word problem. These pictures and models are used to help students write equations for the problem they are solving.
* Instruction includes breaking down word problems into smaller parts. Students use a highlighter to emphasize the important information in the word problem. Also, students paraphrase the word problem so the teacher can determine if the student understands what the question is asking.
* Instruction includes using the 3-Read Protocol to break down the problem. First read the problem to understand the context, possibly without the numbers. Then, read to understand the mathematics. Finally, read to create a plan for solving the problem.

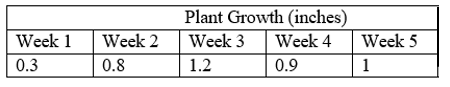
Instructional Tasks

*Instructional Task 1*

Collect 10 used pencils from people in your class. Measure the length of each pencil to the nearest inch and record the lengths on a line plot. What is difference in length of the longest pencil and the shortest pencil?

*Instructional Task 2*

A plant is grown from seed and its change in growth is recorded in a table each week.



Part A. Create a stem-and-leaf plot for the plant growth data.

Part B. How tall is the plant after week 5?

Part C. If the plant begins producing flowers after reaching a height of 12 inches, how many more inches does it need to grow before it flowers?

*Instructional Task 3*

Part A. Explain how you could use a stem-and-leaf plot to find the median of a numerical data set.

Part B. Explain how you could use a stem-and-leaf plot to find the mode of a numerical data set.

Part C. Explain how you could use a stem-and-leaf plot to find the range of a numerical data set.

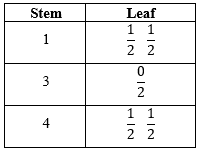
Part D. Use the stem-and-leaf plot below to find the median, mode, and range of the data set.

Ages of Patrons at the Movie Theatre
Stem - 0, Leafs - 6, 6, 7, 7, 7, 8 and 8
Stem - 1, Leafs - 1, 1, 1, 1, 2, 5 and 5
Stem - 2, Leafs - 7, 8, 8, 8 and 9
Stem - 3, Leafs - 2, 2 and 6
Stem - 4, Leafs - 5

Instructional Items

*Instructional Item 1*

The last 5 putt lengths, in feet, for the 18th hole of a golf tournament are shown below.



What is the sum of the 5 putt lengths?

1. 8 feet
2. 9 feet
3. 12 feet
4. 15 feet

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

Appendix A. Summary of Changes

The purpose of this table is to provide a summary of changes from for the latest version of the B1G-M. For questions or feedback on the B1G-M, please direct them to [BESTMath@fldoe.org](mailto:BESTMath@fldoe.org).

|  |  |  |
| --- | --- | --- |
| **Benchmark** | **B1G-M Component** | **Change Made** |
| *MA.4.AR.1.1 & MA.5.AR.1.1* | **Common Misconceptions or Errors** | It is essential to guide students in tackling real-world problems that may provide a quotient with a remainder, where interpretation may be necessary. Foster a mindset that extends beyond the literal content of the word problem. For example, prompt students to consider whether it is plausible to have half of a person/object or if the problem is seeking complete groups. Remind students that interpreting remainders is contingent upon the problem's context and the specific information the problem aims to reveal. Encourage critical thinking and application of mathematical concepts in practical scenarios. |
| *MA.4.AR.1.1 & MA.5.AR.1.1* | **Instructional Tasks** | *Instructional Task 2*  Agusta had $2,385. She spent $729 on a new computer. She wants to spend the rest of her money on different computer accessories. Each accessory cost $65. What is the maximum number of accessories Augusta can buy?   * + 1. 25 accessories     2. 26 accessories     3. 27 accessories     4. 28 accessories |
| *MA.4.AR.1.1 & MA.5.AR.1.1* | **Instructional Items** | *Instructional Item 2 (MTR.3.1)*  Stanley saved $342 last month and $780 this month. He wants to buy as many chairs to place in the auditorium at school as possible. Each chair costs $11. How many chairs can Stanley buy? |
| *MA.4.AR.1.3 & MA.5.AR.1.2* | **Purpose and Instructional Strategies** | * Instruction allows students to build understanding fractions through modeling and the use of manipulatives. * Students are encouraged to express problem-solving solutions through various means, such as objects, illustrations, tables, graphs, and equations (MTR.2.1). When tackling fraction-related challenges, it is advisable to employ diverse models, including but not confined to area models (rectangles), linear models (fraction strips/bars and number lines), and set models (counters). * There should be student progression from solving problems with tangible objects and illustrations to the utilization of algorithms and equations. Students should articulate the relationships between different concepts and representations, while also selecting the most suitable representation based on the specific context or purpose of the question. |
| *MA.4.AR.1.3 & MA.5.AR.1.2* | **Common Misconceptions or Errors** | Encourage students to use their number sense when determining the size of a product. The teacher can ask students what they know about multiplying fractions to determine how the numbers in the problem are related. |
| *MA.4.AR.1.3 & MA.5.AR.1.2* | **Common Misconceptions or Errors** | Students need to apply what they already know about operations with fractions to create action plans to solve multi-step real-world problems involving fractions. |
| *MA.4.AR.1.3 & MA.5.AR.1.2* | **Common Misconceptions or Errors** | With guiding questions, ask students what operation will be needed to solve the problem. Prompt students to tell you what the first action step will be before they solve. If students struggle to find the first step of the problem, encourage them to create a visual model while rereading. |
| *MA.4.AR.1.3 & MA.5.AR.1.2* | **Instructional Tasks** | *Instructional Task 3*  Michelle paints a wall that is 6 feet high and 62feet long. How many square feet does he paint?   * + 1. 372square feet     2. 392square feet     3. 392square feet   393square feet |
| *MA.4.AR.1.3 & MA.5.AR.1.2* | **Instructional Items** | *Instructional Item 2*  Megan has a collection of 80 stickers, and of them feature cars, while of the stickers have animals. The remaining stickers display various types of shapes. How many stickers in Megan’s collection feature shapes? |
| *MA.5.AR.1.3* | **Purpose and Instructional Strategies** | Additionally, students divide a unit fraction by a non-zero whole number (MA.5.FR.2.4) using division concepts learned in (5.NSO.2.2). |
| *MA.5.AR.1.3* | **Purpose and Instructional Strategies** | * The expectation of this benchmark is not for students to use a specific algorithm multiplicative inverse for dividing fractions. Instead, it is encouraged for students to enhance their understanding by drawing and labeling diagrams. Teachers should present real-world scenarios (MTR.7.1) where it is inherent for students to divide a unit fraction by a non-zero whole number or a whole number by a unit fraction. * Instruction includes allowing students to visualize and practice using patterns and structure to divide unit fractions by non-zero whole numbers (*MTR.5.1*). |
| *MA.5.AR.1.3* | **Strategies to Support Tiered Instruction** | If a student struggles to draw a picture of the problem, guide them along by asking what item in the question is being divided, and into how many/what size parts. Encourage and remind students to use pictures or objects. |
| *MA.5.AR.1.3* | **Instructional Task** | *Instructional Task 2 (MTR.6.1, MTR.7.1)*  A string that is 8 feet long is cut into pieces that are feet long. How many pieces are there? |
| *MA.5.AR.1.3* | **Instructional Items** | *Instructional Item 2*  Mr. Smith has a rectangular table with an area of 15 square feet and another square table with an area of 9 square feet. He plans to cover both tables with tiles, where each tile has an area of square foot. How many tiles does he need to completely cover both tables without any gaps or overlaps? |
| *MA.5.AR.1.3* | **Instructional Items** | *Instructional Item 3 (MTR.5.1, MTR.6.1)*  Use operation symbols to make the equation true. |
| *MA.5.AR.2.1* | **Connecting Benchmarks/ Horizontal Alignment** | * MA.5.NSO.2.1, MA.5.NSO.2.2 |
| *MA.5.AR.2.1* | **Purpose and Instructional Strategies** | Students continue to deepen their understanding of operations with whole numbers (MA.5.NSO.2.1/2.2). |
| *MA.5.AR.2.1* | **Purpose and Instructional Strategies** | Explain to students that parentheses make the expression clearer, but they are not necessary when multiplication comes before addition, as seen in the example. |
| *MA.5.AR.2.1* | **Purpose and Instructional Strategies** | Encourage and motivate students to utilize properties to rewrite expressions. Instruction includes a review of the Distributive, Associative, and Commutative Properties of Addition and Multiplication. Reinforce the concept that the Distributive Property involves the distribution of multiplication over addition, while the Commutative Property entails a change in the order of the elements. |
| *MA.5.AR.2.1* | **Common Misconceptions or Errors** | * Students may have confusion with parentheses. Misplacement or omission of parentheses can lead to incorrect interpretations of expressions. Students might forget to use parentheses where necessary or include unnecessary ones, altering the meaning of the expression. * Additional misinterpretations include misunderstanding the order of operations, where a student mistakenly performs an operation over another (e.g. addition before multiplication with no parentheses present). |
| *MA.5.AR.2.1* | **Strategies to Support Tiered Instruction** | Place value disks can be used as a visual representation in addition to the place value chart. |
| *MA.5.AR.2.1* | **Instructional Tasks** | *Instructional Task 3*  Which numerical expression is equivalent to the written expression?   * 1. times the quantity 97 plus 156 |
| *MA.5.AR.2.1* | **Instructional Items** | *Instructional Item 3*  Melissa spends time each evening designing bracelets for her friends. The number of minutes she spends can be represented by the variable . Use the variable to complete the following tasks.   1. Write an algebraic expression to represent the number of minutes Melissa’s friend Gabriella spends designing bracelets in an evening if she designs 45 minutes more that evening than Melissa. 2. Write an algebraic expression to represent the number of minutes Melissa will spend designing bracelets at home in 9 weeks. |
| *MA.5.AR.2.2* | **Purpose and Instructional Strategies** | Students evaluate numerical expressions using whole-number computation skills, including multiplying and dividing multi-digit whole numbers (MA.5.NSO.2.1/2.2). |
| *MA.5.AR.2.2* | **Common Misconceptions or Errors** | For example, when given the expression , guide students to follow the order of operations and simplify the parentheses first. Next, students should recognize that subtraction and addition are the remaining operations, and the expression should be solved reading left to right. |
| *MA.5.AR.2.2* | **Instructional Tasks** | *Instructional Task 3*  Solve the following expression. |
| *MA.5.AR.2.3* | **Strategies to Support Tiered Instruction** | Instruction includes opportunities for students to organize and structure their algebraic tasks. Teachers should encourage students to label and annotate expressions on both side of the equation prior to solving. This structured approach enables students to identify occasions where they can apply the properties of operations and the order of operations for problem-solving. Encourage students to contemplate the operations required to solve the expressions on both sides of the equation (*MTR.1.1*). |
| *MA.5.AR.2.3* | **Instructional Tasks** | *Instructional Task 2*  Part A. Determine if the following equation is true or false.  Part B. Determine if the models and steps shown correctly represent the expressions on both sides of the equation in Part A.  Model of instructional task 2.Model of instructional task 2. |
| *MA.5.AR.2.3* | **Instructional Items** | *Instructional Item 2*  Use the operation signs once in expression below to make the number sentence true.  9 \_\_\_ (7\_\_\_4) \_\_\_6\_\_\_2=34 |
| *MA.5.AR.2.4* | **Purpose and Instructional Strategies** | Students are expected to explain how they can use algebraic equations to solve real-world solutions. |
| *MA.5.AR.2.4* | **Common Misconceptions or Errors** | Students might overlook the fact that solving equations involves the application of inverse operations and the reverse order of operations. It is essential to revisit the concepts of the order of operations, the reverse order of operations, and the significance of inverse operations within the context of equations with students. |
| *MA.5.AR.2.4* | **Instructional Tasks** | *Instructional Task 2*  Solve for in each equation. |
| *MA.5.AR.2.4* | **Instructional Items** | *Instructional Item 2*  Students at Maple Middle School are fundraising for a field trip, and they need to collect a total of $800. Currently, they have $250 from a bake sale. They plan to sell raffle tickets for $5 each to raise more funds. Write an equation to represent the situation, solve for the variable which represents the number of raffle tickets they need to sell to reach their goal, and explain the steps to find the value of |
| *MA.5.AR.3.1* | **Purpose and Instructional Strategies** | In the Grade 4 Accelerated course, the expectation builds on student knowledge of translating numerical expressions into written mathematical descriptions and extends to students writing a rule with one or two operations as an expression. |
| *MA.5.AR.3.1* | **Common Misconceptions or Errors** | Students may focus on a rule that works for a single input and its corresponding output. Remind students that the rule must work for all terms in a table. |
| *MA.5.AR.3.1* | **Common Misconceptions or Errors** | Students may be unfamiliar with the use of parentheses to show multiplication.   * For example, the variable *x* can be any number. The expression 8*x* has different values depending on what *x* is. For example, if *x*=2, the 8 *x* = 8(2)= 8 x 2= 16. If *x*=7, then 8 *x* = 8(7)= 8 x 7= 56. |
| *MA.5.AR.3.1* | **Instructional Tasks** | *Instructional Task 2*  Write an expression that can be a rule for the terms shown below. |
| *MA.5.AR.3.1* | **Instructional Items** | *Instructional Item 2*  Alexis is booking a vacation rental for $50 per night. Write an expression to determine the total cost of renting the vacation home for n nights. What would be the total cost for a 10-night stay? |
| *MA.5.AR.3.2* | **Connecting Benchmarks/ Horizontal Alignment** | MA.5.DP.1.1 |
| *MA.5.AR.3.2* | **Purpose and Instructional Strategies** | Using tables to represent input/output tables prepares students to collect and represent data in line graphs (MA.5.DP.1.1). |
| *MA.5.AR.3.2*  *MA.5.AR.3.2* | **Common Misconceptions or Errors** | When noting values in a two-column table, students may confuse input and output values.   1. For example, some students may incorrectly identify the equations in the output column as the actual outputs. It's important to highlight that the numeric outputs in the tables represent the solutions to the provided equations. 2. The substitution of numeric inputs into a rule helps to determine the corresponding outputs. An input/output table serves as a tool for organizing inputs and their respective outputs. |
| *MA.5.AR.3.2* | **Instructional Items** | *Instructional Item 2*  If each table in the U-shape arrangement can seat 4 people, what is the minimum number of tables required to accommodate a banquet for 40 people? 60 people? 180 people? Create a two-column table to record the inputs and outputs. |
| *MA.4.M.1.1 & MA.5.M.1.1* | **Purpose and Instructional Strategies** | This benchmark should be taught in conjunction with MA.4.M.1.2, where students acquire an understanding of conversion rules (e.g. multiply to change a larger unit to a smaller unit and divide to change a smaller unit to a larger unit.) This foundational concept is applicable to the conversion of any unit of measure. Additionally, in Grade 6 (MA.6.AR.3.5), students further enhance their understanding and establish connections between measurement conversion and the broader context of ratios and rates. During Grade 5, students utilize ratio reasoning to perform unit conversions. The application of ratio reasoning relies on procedures learned through the multiplication and division of fractions, skills that prove valuable in the context of measurement conversion. |
| *MA.4.M.1.1 & MA.5.M.1.1* | **Purpose and Instructional Strategies** | * During instruction, teachers can integrate the relationship between metric units to support student understanding of the place value system. Students utilize their understanding of the patterns of multiplying and dividing with zeros when working with metric conversions. For students to build understanding of metric units, students can list units in order from greatest to least and practice conversion in relation to the place value position. * For students to have a better understanding of measurement conversions and proportional relationships, allow students to have exposure to attributes of items being measured. Students should develop a foundation of the relative size of each unit and should have practice converting from a small unit to a larger unit and from a large unit to a smaller unit. |
| *MA.4.M.1.1 & MA.5.M.1.1* | **Common Misconceptions or Errors** | * Students confuse renaming units of measurements when converting mixed measures.   + For example, when adding 3 feet 5 inches to 4 feet 9 inches, they get 7 feet 14 inches. Students may not understand that 14 inches can be converted into 1 foot 2 inches, for an overall answer of 8 feet 2 inches. |
| *MA.4.M.1.1 & MA.5.M.1.1* | **Common Misconceptions or Errors** | * Students may not know which operation to use when converting measurements. During instruction, teachers can use manipulatives and a real-world approach so that students make connections to units. For example, teachers can use containers that equal cups, pints, quarts, and gallons when giving explicit instruction on units of liquid capacity. Real- world application allows students to have a deeper understanding of the relative size of units that may be abstract. |
| *MA.4.M.1.1 & MA.5.M.1.1* | **Common Misconceptions or Errors** | * A common mistake that students may make when encountering word problems is when problems contain information in different units. Students should convert different units to one common unit before calculating. |
| *MA.4.M.1.1 & MA.5.M.1.1* | **Strategies to Support Tiered Instruction** | * Instruction includes opportunities to connect conversions of metric units to place value positions. Instruction includes utilizing place value mats when computing metric conversions to connect understanding of the relationship between the metric units. Students who struggle with multiplying and dividing across decimal place values should be provided with opportunities. * Instruction includes opportunities to connect conversions of time to fractional parts such as halves and quarters. Instruction allows students to visualize parts of a clock in connection to real-world time application, |
| *MA.5.M.2.1* | **Purpose and Instructional Strategies** | In Grade 6, students are expected to fluently use decimal operations to solve equations with variables (MA.6.NSO.2.3). |
| *MA.5.M.2.1* | **Purpose and Instructional Strategies** | * In this benchmark, money is expressed as decimal numbers, so decimal operations can be used to solve problems involving money. Instruction allows students to make computations flexibly using various strategies *(MTR.2.1)* |
| *MA.5.M.2.1* | **Common Misconceptions or Errors** | * Students may get confused when reading word problems. Teachers should encourage students to draw and label diagrams with monetary value as a visual aid. |
| *MA.5.M.2.1* | **Strategies to Support Tiered Instruction** | Teachers have the opportunity to observe student behaviors as evidence that students are checking for reasonableness while exhibiting proficiency with mathematical thinking and reasoning standards. When assessing for reasonableness students should actively estimate to discover and explore possible solutions to problems. Students should use benchmark quantities to determine if a solution makes sense. Students should also check calculations during and after calculations when solving a problem and they verify possible solutions by explaining the methodology used. When assessing reasonableness, students can analyze results of a problem based on the given context before computation (*MTR.6.1*). |
| *MA.5.M.2.1* | **Strategies to Support Tiered Instruction** | The teacher encourages and guides students to think backwards when completing word problems. Identify what the problem is asking and create action steps to solve the problem. For example, why do you subtract when a question asks you to find how much change will be given back after a purchase? Students should have an understanding that change back is the difference between the amount of money given and the cost of an item.  The teacher encourages students to utilize peer discourse to identify important information when solving word problems. Have students read word problems to a partner and explain to each other what steps are needed to solve the problem (*MTR.4.1*). |
| *MA.5.M.2.1* | **Instructional Tasks** | *Instructional Task 2*  A restaurant owner is buying ice. She needs 60 pounds. Should she buy the 20-pound bags of ice for $4.78 or the 10-pound bags of ice for $2.75 each? |
| *MA.5.M.2.1* | **Instructional Tasks** | *Instructional Task 3*  Ronald goes to dinner with some friends and pays the bill. Each person orders a meal that costs $14.95. Ronald gave the server a $11.96 tip. The total bill, including the tip, was $71.76. How many friends did Ronald pay for? |
| *MA.4.NSO.1.1 & MA.5.NSO.1.1* | **Purpose and Instructional Strategies** | Instruction builds on the patterns of multiplying by 10 and , and extends to multiplying by values such as 100 and that will cause the digits to shift more than one place to the left and right. (MTR.5.1) |
| *MA.4.NSO.1.1 & MA.5.NSO.1.1* | **Instructional Tasks** | *Instructional Task 3*  Leah wrote the following expressions on her paper:  and  Part A Explain how the value of the 7 in 7.4 changes when it is multiplied by 100. Why does this happen?  Part B Explain how the value of the 7 in 7.4 changes when it is multiplied by . Why does this happen? |
| *MA.4.NSO.1.1 & MA.5.NSO.1.1* | **Instructional Items** | *Instructional Item 3*  0.03 is the value of which number?   1. 0.003 2. 0.3 3. 3 4. 300 |
| *MA.4.NSO.1.1 & MA.5.NSO.1.1* | **Instructional Items** | *Instructional Item 4*  The clues below describe the 4 digits of a mystery number that contains the following digits:  2, 5, 7, 9.   * The value of the 2 is the value of 2 in 2, 376. * The value of the 5 is the value of 5 in 549. * The value of the 7 is the value of 7 in 79,428. * The value of the 9 is the value of 9 in 9,823. |
| *MA.4.NSO.1.5 & MA.5.NSO.1.4* | **Instructional Tasks** | *Instructional Task 3 (MTR 3.1)*  List the following numbers in order from least to greatest,  .34, , 23 hundredths, 14 tenths |
| *MA.4.NSO.1.5 & MA.5.NSO.1.4* | **Instructional Items** | *Instructional Item 3*  Order 2.592, 2.59 and 2.399 from least to greatest. |
| *MA.4.NSO.1.5 & MA.5.NSO.1.4* | **Instructional Items** | *Instructional Item 4*  Use the number line to complete the questions below.  number line  Part A: What value does Point A represent?  Part B: Which value does Point C represent?   1. 0.77 2. 0.772 3. 0.779 4. 0.78   Part C: Plot Point B between Points A and C. What is the value of Point B? |
| *MA.4.NSO.1.5 & MA.5.NSO.1.4* | **Instructional Items** | *Instructional Item 5*  Select the comparison symbol to make the statement true .06 .006 .6 |
| *MA.4.NSO.2.3 & MA.5.NSO.2.1* | **Purpose and Instructional Strategies** | A standard algorithm is also generalizable meaning that it works no matter how many digits are involved. |
| *MA.4.NSO.2.3 & MA.5.NSO.2.1* | **Purpose and Instructional Strategies** | * Instruction includes helping students understand what each partial product in a standard algorithm represents.   + For example, when using a standard algorithm to find the product of 376 and 218 students can use their understanding of place value and multiplication to write an equation for each partial product.   Illustration for standard algorithim. |
| *MA.4.NSO.2.3 & MA.5.NSO.2.1* | **Strategies to Support Tiered Instruction** | 500 can be used as a compatible number for 513  Reason for using 500 in the example |
| *MA.4.NSO.2.3 & MA.5.NSO.2.1* | **Instructional Tasks** | *Instructional Task 2*  Select six cards from the set of cards below.  6, 2, 3, 8, 5, 4, 9, 1    Part A. Use your cards to create a four-digit number and a two-digit number. Find the product of your four-digit and two-digit numbers.  Part B. Use your cards to create a three-digit number and another three-digit number. Find the product of your three-digit numbers.  Part C. How are your two products similar? How are they different? |
| *MA.4.NSO.2.3 & MA.5.NSO.2.1* | **Instructional Tasks** | *Instructional Task 3 (MTR.4.1, MTR.5.1)*  Using the digits 2, 3, 6, and 8 arrange them to create two 2-digit numbers that when multiplied, will yield the smallest product. |
| *MA.4.NSO.2.3 & MA.5.NSO.2.1* | **Instructional Items** | *Instructional Item 2*  What is the product of 427 ? |
| *MA.4.NSO.2.3 & MA.5.NSO.2.1* | **Instructional Items** | *Instructional Item 3*  Select the expression that has a product of 255.   1. 15 X 17 2. 25 X 13 3. 25 X 25 4. 75 X 15 |
| *MA.4.NSO.2.4 & 5.NSO.2.2* | **Purpose and Instructional Strategies** | * When used efficiently, partial quotients is a suitable procedure for dividing multi-digit numbers. It can often be more reliable and efficient for students than the long division algorithm. Students can demonstrate fluency by skillfully using partial quotients, and should be able to understand the full long division algorithm.   + For example, students can use place value understanding, estimation and partial quotients to solve Students can begin by determining that the product gets them close to their dividend of 34,681 and continue by subtracting groups of 15 based place value and multiples of 15.Illustration for standard algorithim. |
| *MA.4.NSO.2.4 & 5.NSO.2.2* | **Instructional Tasks** | Existing Task was heavily amended.  *Instructional Task 1 (MTR.7.1)*  The Magnolia Outreach organization is donating 6,924 pounds of rice to families in need. They pour all the rice into 15-pound containers.  Part A. How many containers will they fill completely if they use all the rice?  Part B. How many containers will they need for all of the rice?  Part C. Will the Magnolia Outreach organization completely fill all of the containers needed for all of the rice? If not, will the partially filled container be more or less than half-full? Explain how you know. |
| *MA.4.NSO.2.4 & 5.NSO.2.2* | **Instructional Tasks** | *Instructional Task 2*  Natalie says that 5,978 is not divisible by 24 a whole number of times. Is Natalie correct? Why or why not? |
| *MA.4.NSO.2.4 & 5.NSO.2.2* | **Instructional Items** | *Instructional Item 2*  Part A. What is the quotient of 65?  Part B. Write the remainder of 65, as a fraction. |
| *MA.4.NSO.2.4 & 5.NSO.2.2* | **Instructional Items** | *Instructional Item 3*  What is the quotient of ?   1. 20 2. 21 3. 201 4. 201 |
| *MA.4.NSO.2.7 & MA.5.NSO.2.3* | **Purpose and Instructional Strategies** | A standard algorithm is also generalizable meaning that it works no matter how many digits are involved. |
| *MA.4.NSO.2.7 & MA.5.NSO.2.3* | **Purpose and Instructional Strategies** | Instruction includes explaining that lining up the decimals points to add or subtract decimals also lines up the corresponding place values. Due to the fixed denominator associated with each place value, such as or , the process of adding or subtracting the digits in a specific column is the same as adding or subtracting fractions with like denominators. |
| *MA.4.NSO.2.7 & MA.5.NSO.2.3* | **Purpose and Instructional Strategies** | * Instruction makes the connection to decimal and fraction equivalence when solving addition and subtraction equations involving different numbers of decimal places.   + For example, when solving students can use their understanding of decimal and fraction equivalence to write 7.5 as 7.50 because *seven and five tenths* is equivalent to *seven and fifty hundredths.* |
| *MA.4.NSO.2.7 & MA.5.NSO.2.3* | **Instructional Tasks** | *Instructional Task 2*  A school is packing textbooks into a shipping container. The table below shows the total weights of the textbooks for five different subject areas.  Table for instructional task 2  If the weight limit of the shipping container is 75 pounds, which combination of subject area textbooks will fit in the shipping container?   1. Math, Science and Music 2. ELA, Science and Social Studies 3. Math, ELA and Science 4. Math, Science and Social Studies |
| *MA.4.NSO.2.7 & MA.5.NSO.2.3* | **Instructional Items** | *Instructional Item 2*  An equation is shown. What is the missing value in the equation? |
| *MA.5.NSO.1.2* | **Purpose and Instructional Strategies** | Instruction includes explanation of the word *and* when reading decimals and writing decimals in word form. Students should understand that the word *and* represents the decimal point in the number and separates the portions of the number that are greater than and less than 1. |
| *MA.5.NSO.1.2* | **Instructional Tasks** | *Instructional Task 2*  Micah says that the expanded form of 3.627 is  Jayden says that the expanded form of 3.627 is .  Their teacher explains that they are both correct. Explain why this is the case. |
| *MA.5.NSO.1.2* | **Instructional Items** | *Instructional Item 3*  Which of the following correctly represents 9.023? Select all that apply.   1. *Nine and twenty-three hundredths* 2. *Nine and twenty-three thousandths* |
| *MA.5.NSO.1.3* | **Instructional Items** | *Instructional Item 2*  What number is composed of and ? |
| *MA.5.NSO.1.5* | **Common Misconceptions or Errors** | * Students may confuse the place value to which they are rounding a number.   + For example, when rounding 29.834 to the nearest tenth, they may have difficulty determining that 29.834 is between 29.8 and 29.9. The reliance on mnemonics, songs or rhymes during instruction can often confuse students further because they may lack the conceptual understanding for rounding decimals. |
| *MA.5.NSO.1.5* | **Instructional Items** | *Instructional Item 4*  For the number shown below, select all of the values for the missing digit that would cause 47.63 to round to 47.64 when round to the nearest hundredth.  47.63   1. 2 2. 4 3. 5 4. 6 5. 8 |
| *MA.5.NSO.1.5* | **Instructional Items** | *Instructional Item 5*  Which of the following numbers rounds to 84.5 when rounded to the nearest tenth?  a. 84.42  b. 84.437  c. 84.526  d. 84.55 |
| *MA.5.NSO.2.4* | **Purpose and Instructional Strategies** | Students should have practice solving multiplication and division equations with decimals using strategies based on place value and the properties of operations. |
| *MA.5.NSO.2.4* | **Purpose and Instructional Strategies** | * Instruction includes dividing decimals as if all of the numbers were whole numbers and then using estimation to place the decimal point.   + For example, to solve a reasonable estimate of 12 could be found by determining how many groups of 3 *tenths* can be formed from the 36 *tenths* in 3.69. Dividing 3.69 by 0.3 as if they are whole numbers would result in a quotient of 123. The estimate of 12 would be used to place the decimal after the 2 in 123 resulting in a quotient of 12.3 for |
| *MA.5.NSO.2.4* | **Purpose and Instructional Strategies** | * Instruction makes the connection between multiplication of fractions and decimals to help students reason about the magnitude of their products and determine correct placement of the decimal point.   + For example, can be written as to make connections to multiplying fractions and help students understand why their product results in a decimal to the thousandths. For this problem, so = 0.052. |
| *MA.5.NSO.2.4* | **Strategies to Support Tiered Instruction** | * Instruction includes opportunities to predict and explain the relative size of the quotient of a decimal and a whole number or two decimals. Students use models based on place value to check their prediction and solve. The teacher guides students to connect their quotients to the number of groups or size of the groups based on the magnitude of the divisor and dividend.   + For example, students can use a decimal grid to solve by using the following model:   **Decimal grid for example above.** |
| *MA.5.NSO.2.4* | **Strategies to Support Tiered Instruction** | * + For example, students can use a decimal grid to solve 0.4 by using the following model:   Decimal graph for example above.   * + For example, students can use a decimal grid to solve 0.54 by using the following model:   Decimal grid for example above. |
| *MA.5.NSO.2.4* | **Instructional Items** | *Instructional Item 2*  What is the quotient of ? |
| *MA.5.NSO.2.4* | **Instructional Items** | *Instructional Item 3*  What is the product of ? |
| *MA.5.NSO.2.4* | **Instructional Items** | *Instructional Item 4*  What is the quotient of 3.68? |
| *MA.5.NSO.2.4* | **Instructional Items** | *Instructional Item 5*  Which of the following equations has a missing value of 4.8? Select all that apply.   1. \_\_\_\_\_ 2. \_\_\_\_\_ 3. \_\_\_\_\_ 4. \_\_\_\_\_\_ 5. \_\_\_\_\_\_ |
| *MA.4.FR.1.2* | **Purpose and Instructional Strategies** | * To help students understand decimal fractions, instruction can use models as seen above and name the value using the fraction representation to then extend the place value chart including the decimal point, tenths, and hundredths.   + For example, one whole divided into ten equal parts creates the tenths place and tenths divided into ten equal parts results in hundredths. This is an opportunity to make connections to MA.4.NSO.1.1. |
| *MA.4.FR.1.2* | **Instructional Tasks** | *Instructional Task 2*  Part A  Write each fraction using decimal notation.  \_\_\_\_\_\_\_\_ \_\_\_\_ \_\_\_\_  Part B  Which decimal fractions in Part A are equivalent? Explain your thinking using words or models. |
| *MA.4.FR.1.2* | **Instructional Item** | *Instructional Item 2*  What is the value in fraction form of 3.2? |
| *MA.4.FR.1.2* | **Instructional Item** | *Instructional Item 3*  Table with decimals to math to fraction or mixed number. Decimals - 0.06, 0.6 and 1.6 Fractions 6/10, 6/100 and 1 and 6/10Match each decimal to its representation as a fraction or mixed number. |
| *MA.4.FR.2.4 & MA.5.FR.2.2* | Strategies to Support Tiered Instruction | * Instruction includes real-world situations aligned with the content. The teacher provides a multiplication expression with real-world context and items to represent the situation to make connections.   + For example, the teacher provides the student with the following situation: “The theater class has to make costumes that require foot of ribbon. They need to make 8 costumes. How many feet of ribbon will they need total?” The teacher provides ribbon already cut into foot pieces and has students model the problem. Students then write out the repeated addition problem to match or 2 feet.   + For example, the teacher provides the student with the following situation: “The theater class has 8 feet of ribbon. They need to make 4 costumes and they want to use up all of the ribbon. How many feet of ribbon does each costume use?”The teacher will provide the student with a piece of ribbon 8 feet long and have them mark it off into 4 equal sections and find the length of 1 section (2 feet). * Teacher provides models that represent multiplication expressions to represent the Commutative Property of Multiplication and has students explain the difference in meaning.   + For example, the teacher provides the student with the following models.     Fraction model for example above. Fraction model for example above.   * The teacher provides models and asks students which expression it models.   + For example, the teacher provides students with the following model and asks if it represents or .   Fraction model for example above. |
| *MA.4.FR.2.4 & MA.5.FR.2.2* | **Instructional Tasks** | *Instructional Task 2*  Draw a model to show  Then find the product. If possible, write the product as a mixed number. |
| *MA.4.FR.2.4 & MA.5.FR.2.2* | **Instructional Items** | *Instructional Item 2*  *The model shows*  Fraction model for example above.Fraction model for example above.Fraction model for example above.  What is the product? |
| *MA.4.NSO.2.6* | **Instructional Tasks** | *Instructional Task 2 (MTR.4.1)*  Robert says that 5.21 is equivalent to 5.210. James says that they are not equivalent. Who is correct? Explain how you know. |
| *MA.4.NSO.2.6* | **Instructional Items** | *Instructional Item 2*  What is the equivalent decimal to 4.2?  What is one tenth less than 3?  What is one hundredth more than 4.23?  What is one hundredth less than 2.80? |
| *MA.4.NSO.2.7 & MA.5.NSO.2.3* | **Instructional Tasks** | *Instructional Task 3 (MTR.7.1)*  Anna’s backpack weighs 10.7 pounds. She added her math book to her backpack, and it now weighs 17.3 pounds. How does her math book weigh? |
| *MA.4.NSO.2.7 & MA.5.NSO.2.3* | **Instructional Items** | *Instructional Item 3*  **Table with column headers: 49.18, 12.42, 7.11 and 84.24 Row Headers: 3.12 + 3.99, 45.22 - 32.8, 78.21 + 6.03 and 56.98 - 7.8 1st row of answers:  A, B, C and D 2nd row answers: E, F, G and H 3rd row answers: I, J, K and L 4th row answers: M, N, O and P** |
| *MA.5.NSO.2.5* | **Purpose and Instructional Strategies** | Instruction emphasizes that the decimal point indicates the transition from the ones to the tenths place and that the digits shift when multiplying or dividing by 0.1 or 0.01 *(MTR.2.1, MTR.4.1,MTR.5.1)*.  Place value chart for example above. |
| *MA.5.NSO.2.5* | **Instructional Tasks** | *Instructional Task 1 (MTR.7.1)*  Part A. What is times 15?  Part B. How many dimes are in $1.50?  Part C. Write a division equation to represent how many dimes are in $1.50.  Part D. Write a related multiplication equation to represent how many dimes are in $1.50. |
| *MA.5.NSO.2.5* | **Instructional Tasks** | *Instructional Task 2 (MTR.7.1)*  Part A. What is times 60?  Part B. How many pennies are in $0.60?  Part C. Write a division equation to represent how many pennies are in $0.60.  Part D. Write a related multiplication equation to represent how many pennies are in $0.60. |